Single-Particle Transition (Weisskopf Estimate)

For an electric single-particle transition we assume excitation of only one proton in an average central potential that changes its orbit from j_i to j_f. A magnetic transition takes place when the intrinsic spin is flipped, i.e., $j_i = \ell_i \pm 1/2 \rightarrow j_f = \ell_f \mp 1/2$, respectively. For a magnetic transition the multipolarity λ is given by $|j_i - j_f| = \lambda$ and $|\ell_i - \ell_f| = \lambda - 1$.

A useful scale of $B(E\lambda)$- and $B(M\lambda)$-values are the Weisskopf units which allow a rough estimate of the number of nucleons contributing to radiation. For a transition from an excited state I_i to the ground state I_{gs} we find in the electric (Eλ) and magnetic (Mλ) case

$$B(E\lambda; I_i \rightarrow I_{gs}) = \frac{(1.2)^{2\lambda}}{4\pi}(\frac{3}{\lambda+3})^2 A^{2\lambda/3} \quad e^2(fm)^{2\lambda} \tag{1}$$

$$B(M\lambda; I_i \rightarrow I_{gs}) = \frac{10}{\pi}(1.2)^{2\lambda-2}(\frac{3}{\lambda+3})^2 A^{(2\lambda-2)/3} \quad \mu_N^2(fm)^{2\lambda-2} \tag{2}$$

For the first few values of λ, the Weisskopf estimates are

$$B(E1; I_i \rightarrow I_{gs}) = 6.446\ 10^{-4}\ A^{2/3} \quad e^2(barn) \tag{3}$$

$$B(E2; I_i \rightarrow I_{gs}) = 5.940\ 10^{-6}\ A^{4/3} \quad e^2(barn)^2 \tag{4}$$

$$B(E3; I_i \rightarrow I_{gs}) = 5.940\ 10^{-8}\ A^{2} \quad e^2(barn)^3 \tag{5}$$

$$B(E4; I_i \rightarrow I_{gs}) = 6.285\ 10^{-10}\ A^{8/3} \quad e^2(barn)^4 \tag{6}$$

$$B(M1; I_i \rightarrow I_{gs}) = 1.790 \quad (\frac{e\hbar}{2Mc})^2 \tag{7}$$

Table 1

Coulomb barrier

$$E_B = \frac{Z_a Z_A e^2}{r_B}\left(1 - \frac{a}{r_B}\right)$$

$$r_B = (1.07(A_a^{1/3} + A_A^{1/3}) + 2.72)\,\mathrm{fm}$$

$$a = 0.63\ \mathrm{fm}$$

$$\varepsilon = \hbar\omega/2\pi$$

$$\hbar\omega = \left(\frac{\hbar^2}{m_{aA}}\ \frac{E_B}{a r_B}\left(1 - \frac{a}{r_B}\right)\right)^{\frac{1}{2}}$$

$$m_{aA} = m_a m_A/(m_a + m_A)$$

Inelastic scattering and Coulomb excitation formfactors

$$\left(F_\lambda^{(i)}(r_B)\right)_v = \frac{\sigma_\lambda}{r_B} E_B \left(1 - \frac{3}{2\lambda+1}\left(\frac{R_i}{r_B}\right)^{\lambda-1}\right)$$

$$\sigma_\lambda = \frac{\beta_\lambda R_i}{\sqrt{4\pi}}, \quad R_i = 1.2\ A^{1/3}\mathrm{fm} \qquad (i=a,A)$$

$$\beta_\lambda^2 = \frac{4\pi(2\lambda+1)}{Z^2(3+\lambda)^2}\ \frac{B(E\lambda)}{B_{sp}}$$

The γ-ray strengths listed in Tables III through XIV are expressed in Weisskopf units,[28] based on a nuclear-radius constant $r_0 = 1.20$ fm. The Weisskopf estimates Γ_W (in electron volts) are given below as a function of E_γ (in mega-electron volts) and A:

$$\Gamma_W(E1) = 6.8 \times 10^{-2} A^{2/3} E_\gamma^3$$
$$\Gamma_W(E2) = 4.9 \times 10^{-8} A^{4/3} E_\gamma^5$$
$$\Gamma_W(E3) = 2.3 \times 10^{-14} A^2 E_\gamma^7$$
$$\Gamma_W(E4) = 6.8 \times 10^{-21} A^{8/3} E_\gamma^9$$
$$\Gamma_W(E5) = 1.6 \times 10^{-27} A^{10/3} E_\gamma^{11}$$

$$\Gamma_W(M1) = 2.1 \times 10^{-2} E_\gamma^3$$
$$\Gamma_W(M2) = 1.5 \times 10^{-8} A^{2/3} E_\gamma^5$$
$$\Gamma_W(M3) = 6.8 \times 10^{-15} A^{4/3} E_\gamma^7$$
$$\Gamma_W(M4) = 2.1 \times 10^{-21} A^2 E_\gamma^9$$
$$\Gamma_W(M5) = 4.9 \times 10^{-28} A^{8/3} E_\gamma^{11}$$

The strengths of $E0$ transitions (Table II) are expressed in Wilkinson units (Wi.u.). The definition of

C: Coulomb Scattering

$$\varepsilon = \frac{E_{lab}}{A_1} = \frac{E_{CM}}{A_{12}} = \frac{m v_\infty^2}{2}$$

1. Useful numbers and formulae

(The numerical coefficients in (C.1–3) are for ε in MeV)

$$e^2 = 1.44 \text{ MeV fm}; \quad \frac{\hbar^2}{2m} = 20.9 \text{ MeV fm}^2,$$

a.m.u. (nucleon mass)

$$a = \frac{1}{2} D(\pi) = \frac{e^2}{2} \frac{Z_1 Z_2}{A_{12}\varepsilon} = 0.72 \frac{Z_1 Z_2}{A_{12}\varepsilon} \text{ fm} \tag{C.1}$$

$$k = (\lambda\!\!\!^{-})^{-1} = \left(\frac{\hbar^2}{2m}\right)^{-1/2} A_{12}\varepsilon^{1/2} = 0.219\, A_{12}\varepsilon^{1/2} \text{ fm}^{-1}. \tag{C.2}$$

$$n = ka = \frac{e^2}{2}\left(\frac{\hbar^2}{2m}\right)^{-1/2} \frac{Z_1 Z_2}{\varepsilon^{1/2}} = 0.157 \frac{Z_1 Z_2}{\varepsilon^{1/2}}. \tag{C.3}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_C = \frac{a^2}{4} \sin^{-4}\left(\frac{\Theta}{2}\right). \tag{C.4}$$

$$\varepsilon_C = \frac{E_C}{A_{12}} = \frac{Z_1 Z_2 e^2}{A_{12} R_C}. \tag{C.5}$$

$$= 2 \arcsin \left(\frac{1}{2\frac{\varepsilon}{\varepsilon_C} - 1}\right)$$

$$\Theta_{gr} = 2 \text{ arc sin}\left(\frac{\varepsilon_C}{2\varepsilon - \varepsilon_C}\right) = 2 \text{ arc sin}\left(\frac{a}{R_C - a}\right). \tag{C.6}$$

$$L_{gr} = k R_C \left(\frac{\varepsilon - \varepsilon_C}{\varepsilon}\right)^{1/2} = \left(\frac{\hbar^2}{2m}\right)^{-1/2} A_{12} R_C (\varepsilon - \varepsilon_C)^{1/2}. \tag{C.7}$$

$$\sigma_R = \pi \lambda\!\!\!^{-2} L_{gr}^2 = \pi R_C^2 \left(\frac{\varepsilon - \varepsilon_C}{\varepsilon}\right). \tag{C.8}$$

$$R_C \approx [1.12\,(A_1^{1/3} + A_2^{1/3}) - 0.94\,(A_1^{-1/3} + A_2^{-1/3}) + 3] \text{ fm}. \tag{C.9}$$

2. Relationship between kinematic variables

Required quantity	Formula in terms of D	Formula in terms of L	Formula in terms of Θ
distance of closest approach D		$a\left[1 + \left(1 + \frac{L^2}{n^2}\right)^{\frac{1}{2}}\right]$	$a\left[1 + \sin^{-1}\left(\frac{\Theta}{2}\right)\right]$
angular momentum L	$n\left[\frac{D}{a}\left(\frac{D}{a} - 2\right)\right]^{\frac{1}{2}}$		$n \cot\left(\frac{\Theta}{2}\right)$
scattering angle Θ	$2 \sin^{-1}\left(\frac{a}{D - a}\right)$	$2 \tan^{-1}\left(\frac{n}{L}\right)$	

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = 1.296 \left(\frac{Z_1 Z_2}{E_{CM}}\right)^2 \left(\frac{M_1 + M_2}{M_2}\right)^2 \frac{1}{\sin^4(\theta/2)} \quad \text{mb/sr}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{lab} = 1.296 \left(\frac{Z_1 Z_2}{E_{lab}}\right)^2 \left[\frac{1}{\sin^4(\psi/2)} - 2\left(\frac{M_1}{M_2}\right)^2 + \ldots\right]$$