Refractive index perturbations – Casimir, Unruh, or Hawking?

Ralf Schützhold

Fakultät für Physik

Universität Duisburg-Essen

UNIVERSITÄT DUISBURG ESSEN

Introduction

Kerr effect in non-linear dielectrics

$$n = n_0 + n_2 I$$

E.g., fused silica $n_2 = 3 \times 10^{-16}\,\mathrm{W}^{-1}\mathrm{cm}^2$

$\longrightarrow$ generate refractive index perturbation via $I(t, \boldsymbol{r})$

$$n(t, \boldsymbol{r}) = n_0 + \delta n(t, \boldsymbol{r})\,, \qquad \delta n \ll 1$$

$\longrightarrow$ create photon pairs out of vacuum

- assume zero temperature
- omit photon polarizations
- neglect dispersion $n(\omega)$
- perturbation theory in $\delta n \ll 1$

$$\mathcal{L} = \frac{1}{2}\left(\varepsilon E^2 - B^2\right) = \frac{1}{2}\left([n_0 + \delta n]^2 E^2 - B^2\right)$$

Perturbation Theory

$$\hat{H} = \frac{1}{2}\int d^3r \left(\frac{\hat{D}^2}{[n_0+\delta n]^2} + \hat{B}^2\right) = \hat{H}_0 + \hat{H}_1$$

Interaction picture

$$|\Psi_{\rm out}\rangle = |0\rangle - i\int dt\, \hat{H}_1(t)\,|0\rangle + \mathcal{O}(\delta n^2)$$

Creation of photon pairs

$$|\Psi_{\rm out}\rangle = |0\rangle + \sum_{\boldsymbol{k},\boldsymbol{k'}} \mathcal{A}_{\boldsymbol{k},\boldsymbol{k'}}\,|\boldsymbol{k},\boldsymbol{k'}\rangle + \mathcal{O}(\delta n^2)$$

Two-photon amplitude/probability

$$\mathcal{P}_{\boldsymbol{k},\boldsymbol{k'}} = \left|\mathcal{A}_{\boldsymbol{k},\boldsymbol{k'}}\right|^2 = \frac{\omega_k\omega_k'}{n_0^6}\left|\widetilde{\delta n}(\omega_k+\omega_k',\boldsymbol{k}+\boldsymbol{k'})\right|^2$$

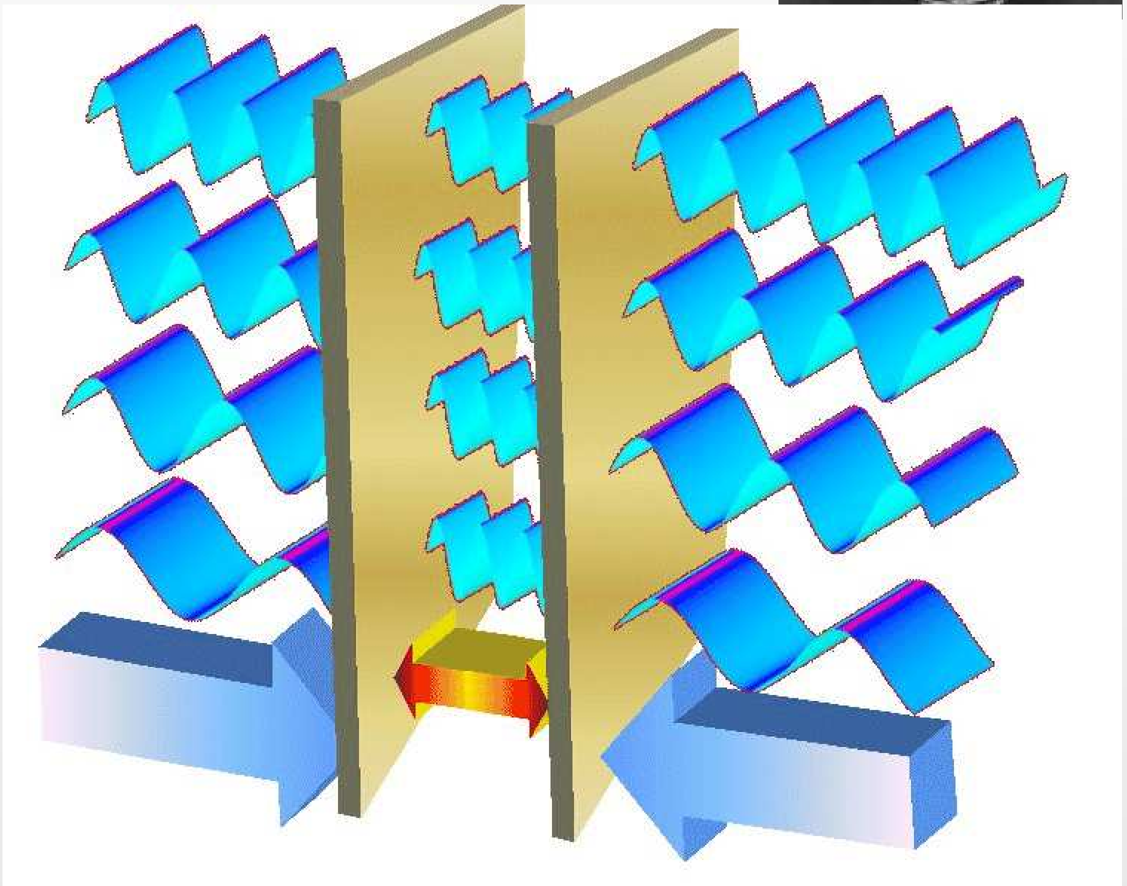
R. S., G. Plunien and G. Soff, Phys. Rev. A **58**, 1783 (1998)

Dynamical Casimir Effect

Static Casimir effect:
Attraction/repulsion in QED vacuum

Dynamical Casimir effect:
Creation of photon pairs out of ground state induced by time-dependent external/classical perturbations:

a) cavity
b) single mirror
c) dielectrics?



(or just boundary conditions?)

Single One-Parameter Pulse

$$\delta n(t, \boldsymbol{r}) = \delta\bar{n}\, f(\Omega t, \Omega \boldsymbol{r}/c)$$

Well behaved function $f = \mathcal{O}(1)$ (e.g., Gaussian)

$\rightarrow$ total emission probability independent of Ω

$$P = \sum_{\boldsymbol{k},\boldsymbol{k}'} \left|\mathcal{A}_{\boldsymbol{k},\boldsymbol{k}'}\right|^2 = \mathcal{O}(\delta n^2) \ll 1$$

Typical photon energy

$$E = \frac{1}{P} \sum_{\boldsymbol{k},\boldsymbol{k}'} \left|\mathcal{A}_{\boldsymbol{k},\boldsymbol{k}'}\right|^2 \omega_k = \mathcal{O}(\Omega)$$

Two-parameter pulse?

Single Two-Parameter Pulse

$$\delta n(t, \boldsymbol{r}) = \delta\bar{n}\, f(\Omega_1 t, \Omega_2 \boldsymbol{r}/c)$$

Two limiting cases

- $\Omega_1 \ll \Omega_2 \rightarrow$ point-like pulse
- $\Omega_1 \gg \Omega_2 \rightarrow$ "cosmological particle creation"

Point-like pulse: expected emitted energy

$$\langle \hat{E} \rangle \propto \int dt \left(\frac{d^4}{dt^4} \left[\int d^3 r\, \delta n(t, \boldsymbol{r}) \right] \right)^2$$

R. S., G. Plunien and G. Soff, Phys. Rev. A **58**, 1783 (1998)

Typical photon energy and total probability

$$E = \mathcal{O}(\Omega_1) \rightsquigarrow P = \mathcal{O}\left(\frac{\Omega_1^6}{\Omega_2^6}\, \delta n^2 \right) \lll 1$$

$\rightarrow$ extremely small effect

Cosmological Particle Creation

$$\delta n(t, \boldsymbol{r}) = \delta \bar{n}\, f(\Omega_1 t, \Omega_2 \boldsymbol{r}/c)$$

For $\Omega_1 \gg \Omega_2$ we may approximate $\delta n(t, \boldsymbol{r}) \approx \delta n(t)$

$$\mathcal{L} \approx \frac{1}{2}\left(\varepsilon(t) E^2 - B^2\right) = \frac{1}{2}\left(n^2(t) E^2 - B^2\right)$$

Analogous to expanding/contracting universe

- pairs of photons with $\boldsymbol{k}' \approx -\boldsymbol{k}$ (entanglement)
- typical photon energy $E = \mathcal{O}(\Omega_1)$!!!
- total probability volume enhancement!

$$P = \mathcal{O}\left(\frac{\Omega_1^3}{\Omega_2^3}\, \delta n^2\right) \overset{?}{\not\ll} 1$$

Moving Pulse

$$\delta n(t,\boldsymbol{r}) = \delta\bar{n}\, f(\Omega[\boldsymbol{r}-\boldsymbol{v}t]/c)$$

First scenario: constant velocity $\boldsymbol{v}$

- $v < c$ no effect (to lowest order in δn)
  "Lorentz" boost $\longrightarrow$ stationary $\delta n(t,\boldsymbol{r}) \to \delta n(\boldsymbol{r})$
- $v > c$ "quantum Cherenkov" radiation
  F. Belgiorno et al, Phys. Rev. Lett. **104**, 140403 (2010)
  R. S., G. Plunien and G. Soff, Phys. Rev. A **58**, 1783 (1998)
  "Lorentz" boost $\longrightarrow$ instantaneous $\delta n(t) \times f(\boldsymbol{r})$

Typical photon energy $E = \mathcal{O}(\Omega)$ and
total emission probability (per unit time)

$$\frac{dP}{dt} = \mathcal{O}\left(\Omega\,\delta n^2\right)$$

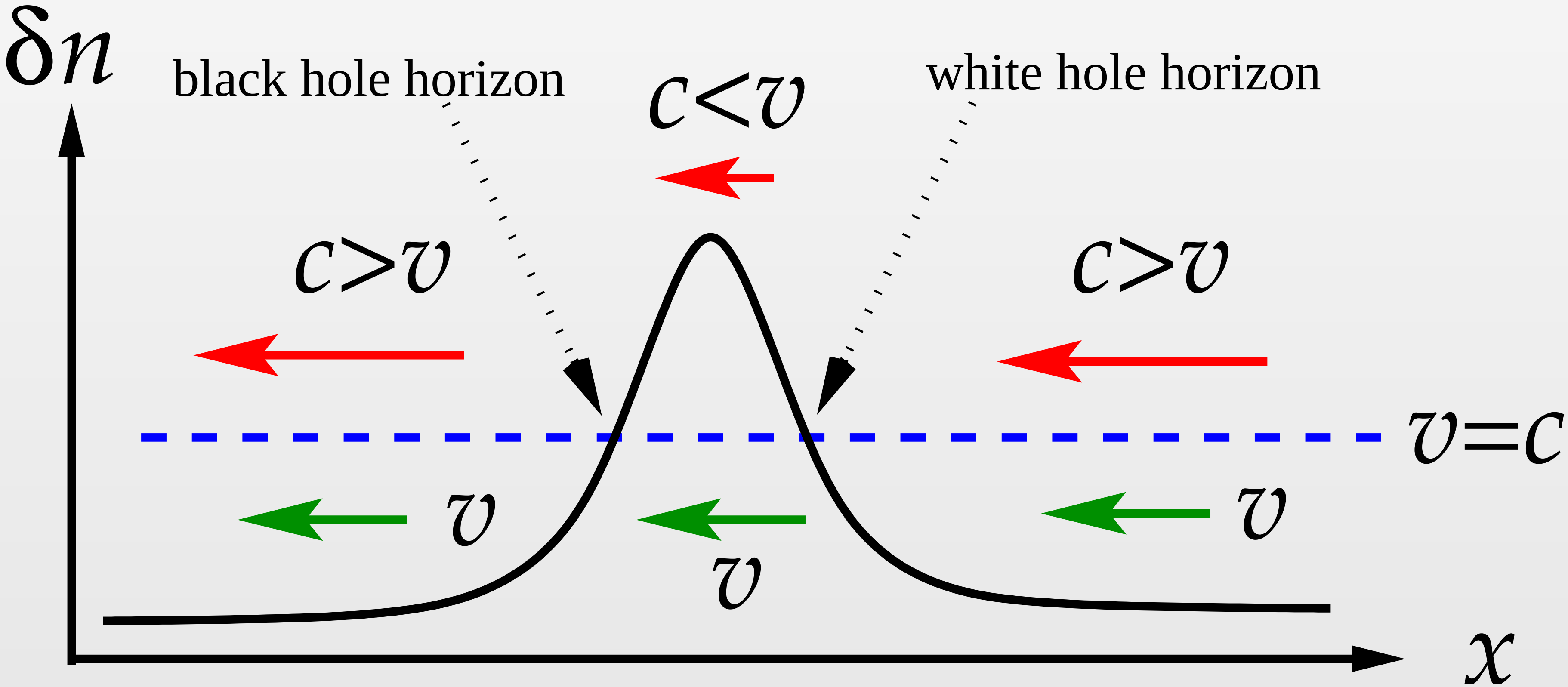
Emission angle $\vartheta = \mathcal{O}(\sqrt{v-c})$ closes for $v \downarrow c$

Hawking Radiation

Fine-tuned pulse velocity

$$\delta n(t,\boldsymbol{r}) = \delta\bar{n}\, f(\Omega[\boldsymbol{r}-\boldsymbol{v}t]/c)\,, \quad c = \frac{1}{n} > v > \frac{1}{n+\delta\bar{n}}$$

Analogue of black and white hole horizon



Various Scenarios. . .

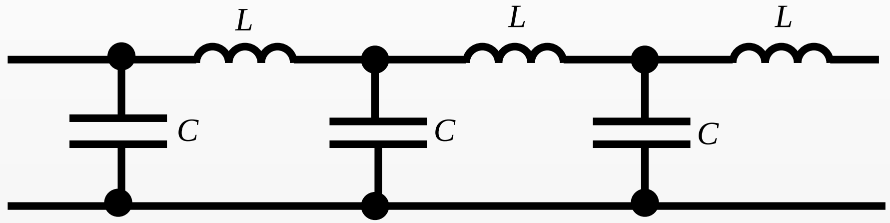
Electromagnetic wave-guide

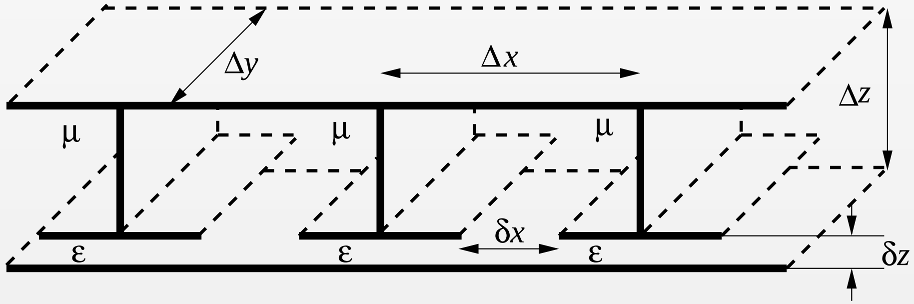
R. S. and W. G. Unruh,

Phys. Rev. Lett. **95**, 031301 (2005)

R. S., G. Plunien and G. Soff,

Phys. Rev. Lett. **88**, 061101 (2002)





Optical fibre

T. G. Philbin et al, Science **319**, 1367 (2008)

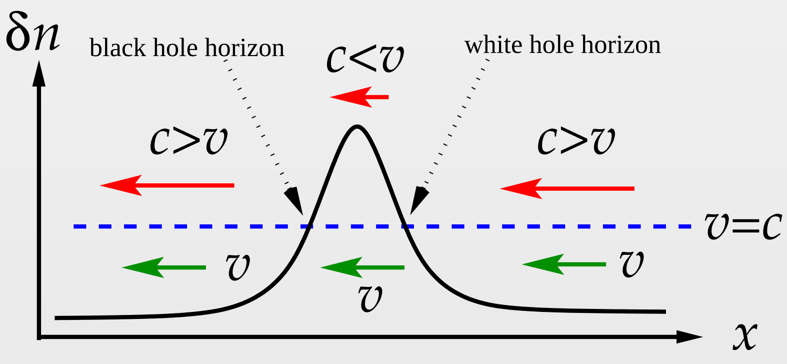
Bulk dielectric

F. Belgiorno et al,

Phys. Rev. Lett. **105**, 203901 (2010)

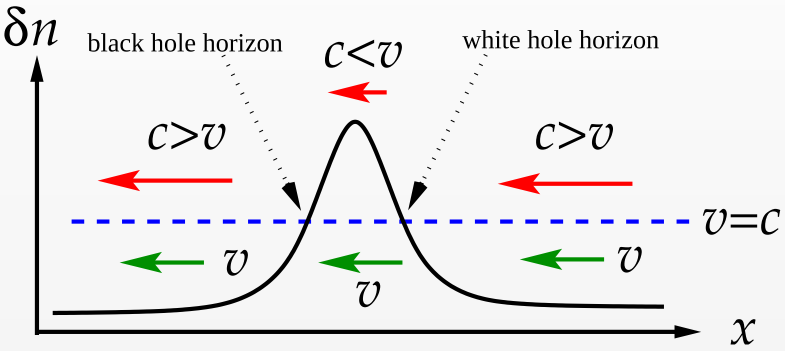
R. S. and W. G. Unruh,

arXiv:1012.2686 [quant-ph]



Sonic black hole analogues etc.

Hawking Temperature



$$\delta n(t,\boldsymbol{r}) = \delta\bar{n}\, f(\Omega[\boldsymbol{r}-\boldsymbol{v}t]/c)\,, \quad c = \frac{1}{n} > v > \frac{1}{n+\delta\bar{n}}$$

Hawking temperature (co-moving frame)

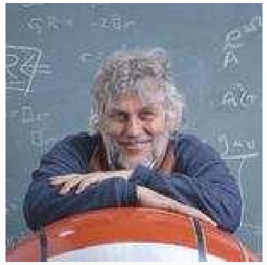
$$T_{\text{Hawking}} \propto \frac{\partial}{\partial x}\delta n \sim \Omega\,\delta n$$

Total emission probability (per unit time) in 3D

$$\frac{dP}{dt} = \mathcal{O}\left(A\,T_{\text{Hawking}}^3\right) \to \mathcal{O}\left(\Omega\,\delta n^3\right)$$

$\to$ beyond perturbation theory 1D: $\mathcal{O}\left(\Omega\,\delta n\right)$

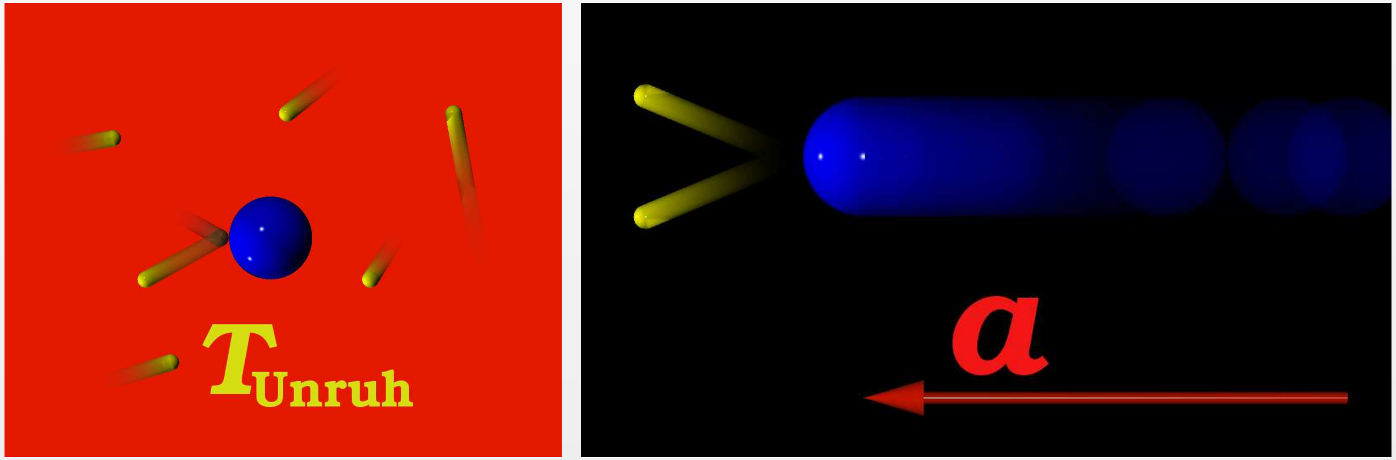
Non-Inertial Pulse



E.g., uniform acceleration → Unruh effect

$$\delta n(t,\boldsymbol{r}) = \delta\bar{n}\, f(\Omega[\boldsymbol{r}-\boldsymbol{r}_P(t)]/c)$$

Scattering in accelerated frame (thermal bath)



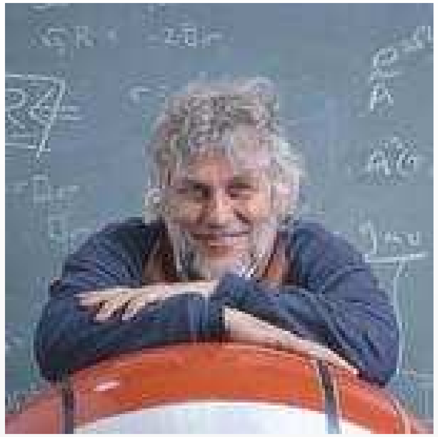
Translation back into inertial (laboratory) frame

$$\frac{dP}{dt} = \mathcal{O}\left(\sigma_{\text{scattering}} T^3_{\text{Unruh}}\right) \to \mathcal{O}\left(\frac{\delta n^2}{\Omega^2}\,\ddot{\boldsymbol{r}}^3_P\right)$$

R. S., G. Schaller and D. Habs, Phys. Rev. Lett. **97**, 121302 (2006);

Phys. Rev. Lett. **100**, 091301 (2008)

Unruh Effect



$$T_{\text{Unruh}} = \frac{\hbar}{2\pi k_{\text{B}} c}\, a = \frac{\hbar c}{2\pi k_{\text{B}}} \frac{1}{d_{\text{horizon}}}$$

Uniformly accelerated detector experiences inertial vacuum state as thermal bath with Unruh temperature

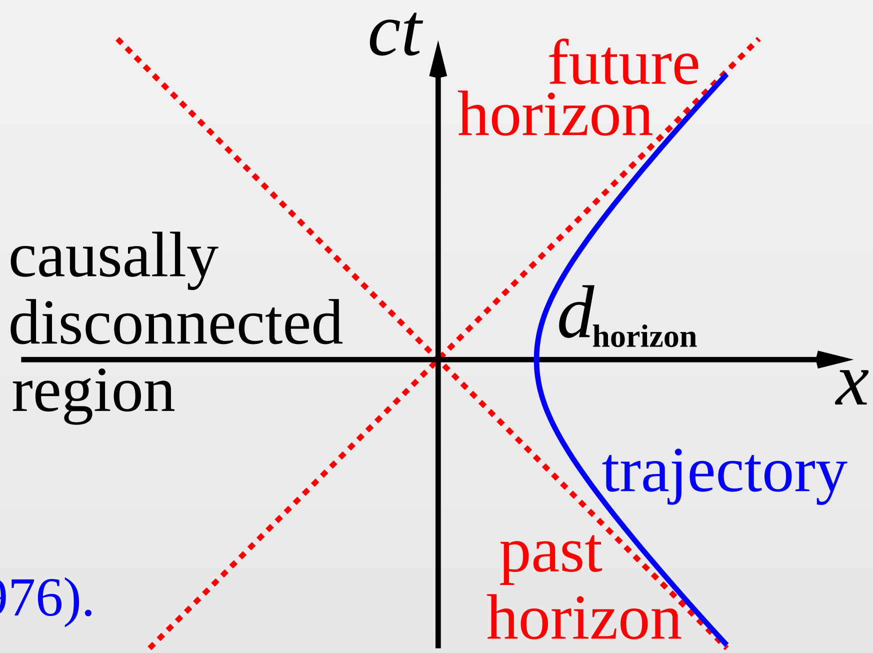
E.g., $a = 9.81\,\text{m/s}^2 \rightsquigarrow T_{\text{Unruh}} \approx 4 \times 10^{-20}\,\text{K}$

Trajectory $x(t)$ with constant acceleration a

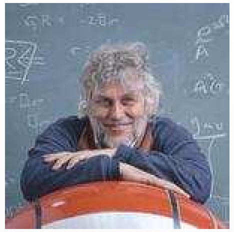
Loss of causal connection

$\longrightarrow$ horizon size

$$d_{\text{horizon}} = c^2/a$$



W. G. Unruh, Phys. Rev. D **14**, 870 (1976).


Summary

Time-dependent refractive index perturbations

$$n(t,\boldsymbol{r}) = n_0 + n_2 I = n_0 + \delta n(t,\boldsymbol{r})\,, \qquad \delta n \ll 1$$

- Dynamical Casimir effect ?
- Cosmological particle creation $n(t)$
- "Quantum Cherenkov" radiation $v > c$
- Hawking radiation $1/n > v > 1/(n+\delta n)$
- Unruh effect $\ddot{\boldsymbol{r}}_P$