Dynamical Casimir-Polder forces

Roberto Passante

DIPARTIMENTO DI FISICA
UNIVERSITA' DEGLI STUDI DI PALERMO
PALERMO, ITALY

in collaboration with*:*
Riccardo Messina, Lucia Rizzuto, Salvatore Spagnolo, Ruggero Vasile

International Workshop on Dynamical Casimir Effect
Padova, Italy, 8 June 2011

Casimir forces are long-range interactions between neutral objects in the vacuum due to their common interaction with the quantum radiation field.

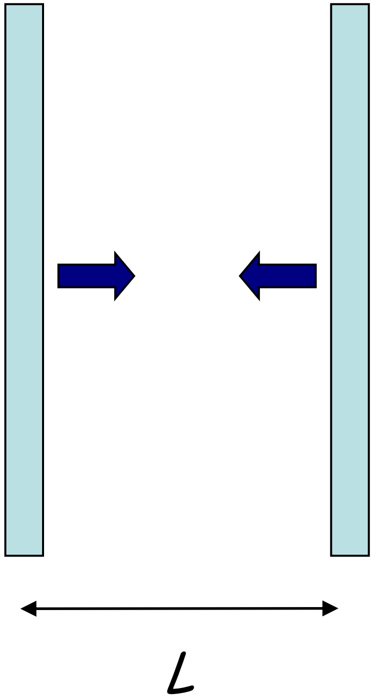
They are an observable manifestation of the quantum zero-point fluctuations of the radiation field, ***even at the macroscopic level.*** Do their existence prove the reality of vacuum fluctuations?

Casimir effect: interactions between neutral macroscopic objects (metallic plates, dielectrics,….)

Casimir-Polder forces: interactions between a neutral atom/molecule or condensate and a conducting or dielectric object

van der Waals/Casimir-Polder forces: long-range interactions between neutral atoms/molecules in the vacuum

Casimir force between two infinite parallel and perfectly conducting plates



Force for unit area: $$\frac{F}{A} = -\frac{\pi^2 \hbar c}{240 L^4}$$

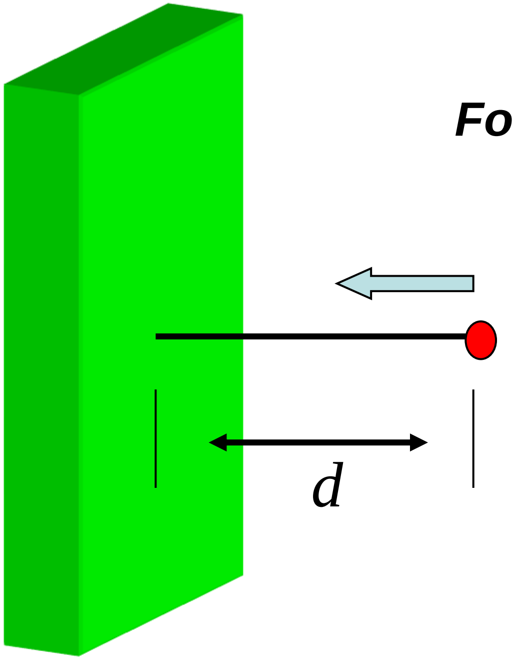
This force can be obtained from the change of the zero-point energy due to the boundary conditions.

Order of magnitude of the Casimir force:

$$A = 1\mathrm{cm}^2,\ L = 1\mu\mathrm{m} \ \rightarrow\ F \approx 10^{-7}\,\mathrm{N}$$

The Casimir force has been measured with remarkable precision (1-10%) for several topologies (wall-wall, wall-sphere, etc).

Casimir-Polder force between a ground-state atom/molecule and an infinite perfectly conducting plate *(static situation)*



Force on the atom:

$$\begin{cases} \sim \dfrac{1}{d^4} & (\text{near zone: } d << c/\omega_o) \\ \sim \dfrac{1}{d^5} & (\text{far zone: } d >> c/\omega_o) \end{cases}$$

In the far zone:

$$F = -\frac{3\hbar c\alpha}{2\pi d^5}$$

(α = static polarizability of the atom,
ω_o = typical atomic transition frequency)

The (electric) atom-wall Casimir-Polder force is attractive for any distance.
The atom-wall Casimir-Polder force has been recently measured with precision, both in the near and in the far zone.

The atom-wall potential energy from the interaction energy of the ground- (excited-) state atom with the vacuum field fluctuations (modified by the presence of the wall)

dressed ground (excited) state

$$\Delta E_{g(e)} = \frac{1}{2}\,{}_D\left\langle \left\{0_{\mathbf{k}j}\right\}\downarrow(\uparrow)\right| H_I \left|\left\{0_{\mathbf{k}j}\right\}\downarrow(\uparrow)\right\rangle_D$$

mode functions

position of the atom

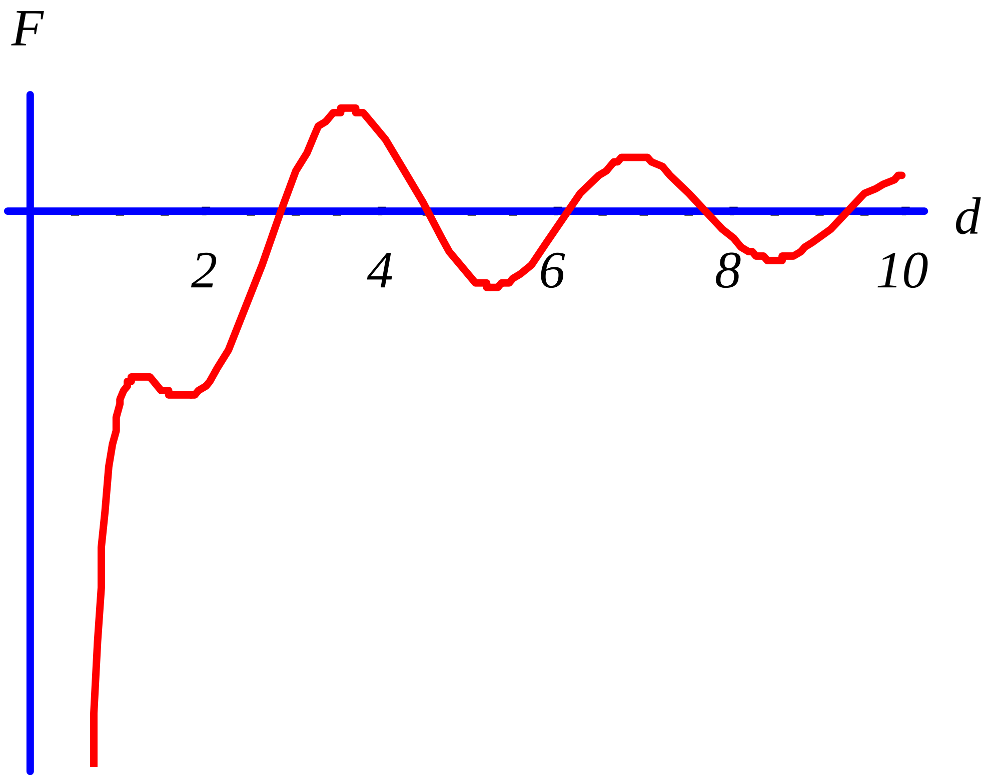
$$H_I = -i\left(\frac{2\pi\hbar}{V}\right)^{1/2}\sum_{\mathbf{k}j}\omega_k^{1/2}\boldsymbol{\mu}\bullet\mathbf{f}_{\mathbf{k}j}\left(\mathbf{r}_A\right)\left(a_{\mathbf{k}j}+a_{\mathbf{k}j}^{\dagger}\right)$$

atomic dipole moment

$$F_{g(e)}(d) = -\frac{\partial}{\partial d}\Delta E_{g(e)}$$

(quasistatic approach)

Static Casimir-Polder force for an excited atom



Negative (positive) values of the force indicate an attractive (repulsive) force. (For a ground-state atom the force is attractive for any distance)

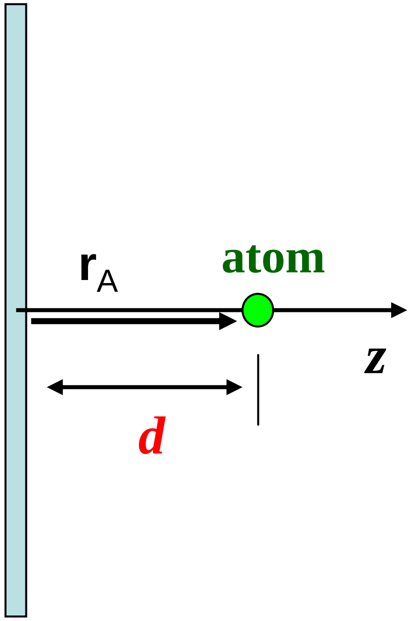
The atom-wall distance is in units of k_0^{-1}

Dynamical (time-dependent) atom-wall Casimir-Polder force for an initially bare atom

Atom-field state at *t* = *0*
(nonequilibrium states)

bare ground state: $\left|\{0_{\mathbf{k}j}\}\downarrow\right\rangle$

bare excited state: $\left|\{0_{\mathbf{k}j}\}\uparrow\right\rangle$



wall at *z* = *0*

→ *atomic dynamics (self-dressing)*

→ *time-dependent Casimir-Polder force*

In this case the dynamics is due to the Hamiltonian evolution of the atom, starting from a nonequilibrium state, and not to some external action on the system.

The time-dependent Casimir-Polder interaction energy

Interaction energy:

$$\Delta E_{g(e)}(t)=\frac{1}{2}\left\langle\left\{0_{\mathbf{k}j}\right\}\downarrow(\uparrow)\right|H_I(t)\left|\left\{0_{\mathbf{k}j}\right\}\downarrow(\uparrow)\right\rangle$$

(Heisenberg representation)

For a two-level atom (solving Heisenberg equation for atomic and field operators at the second order)

$$\Delta E_{g(e)}(t)=-\frac{2\pi}{V}\sum_{\lambda=x,y,z}\mu_\lambda^2\sum_{\mathbf{k}j}\frac{k}{k\pm k_0}\left[\mathbf{f}_{\mathbf{k}j}(\mathbf{r}_A)\right]^2\left(1-\cos\left[(k\pm k_0)t\right]\right)$$

upper (lower) signs are for a ground- (excited-) state atom

According to the relative orientation of the atomic dipole moment to the plate

$$\Delta E_z(d) = -\frac{\mu_z^2}{4\pi d^3}\int_0^\infty dk\,\frac{-2kd\cos(2kd)+\sin(2kd)}{k\pm k_0}\left(1-\cos\left[ct\left(k\pm k_0\right)\right]\right)$$

$$\Delta E_{x/y}(d) = -\frac{\mu_{x/y}^2}{8\pi d^3}\int_0^\infty dk\,\frac{-2kd\cos(2kd)+\left(1-4k^2d^2\right)\sin(2kd)}{k\pm k_0}$$

$$\times\left(1-\cos\left[ct\left(k\pm k_0\right)\right]\right)$$

Atom in its bare ground state at t = 0

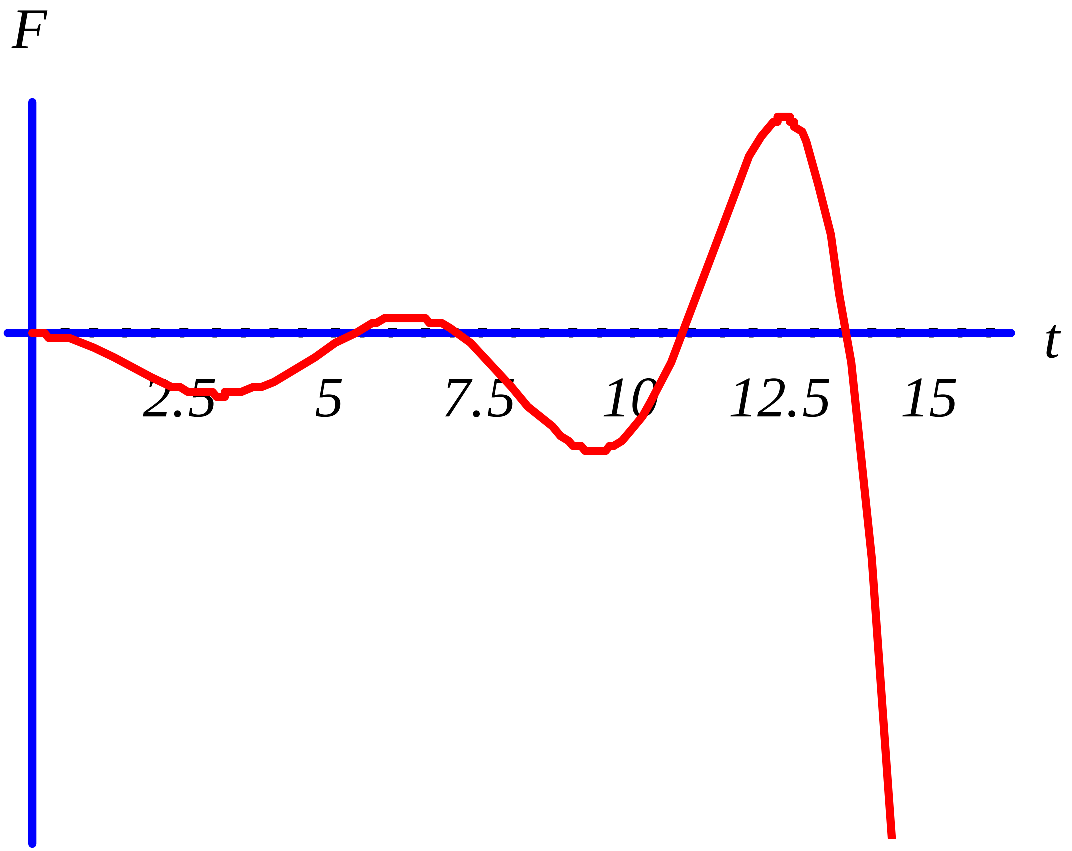
The dynamical Casimir-Polder force is characterized by the timescale $\tau = 2d/c$

(backreaction time: time taken by the field emitted by the atom to go back at the atomic position, after reflection on the conducting wall)

A spike of radiation is emitted by the atom at $t=0$ (singular at the light-cone, due to the dipole approximation): it can interact with the atom after reflection on the wall

Dynamical Casimir-Polder force on the initially bare ground-state (isotropic) atom, as a function of time ($c = 1, k_0 = 1, d = 10$)

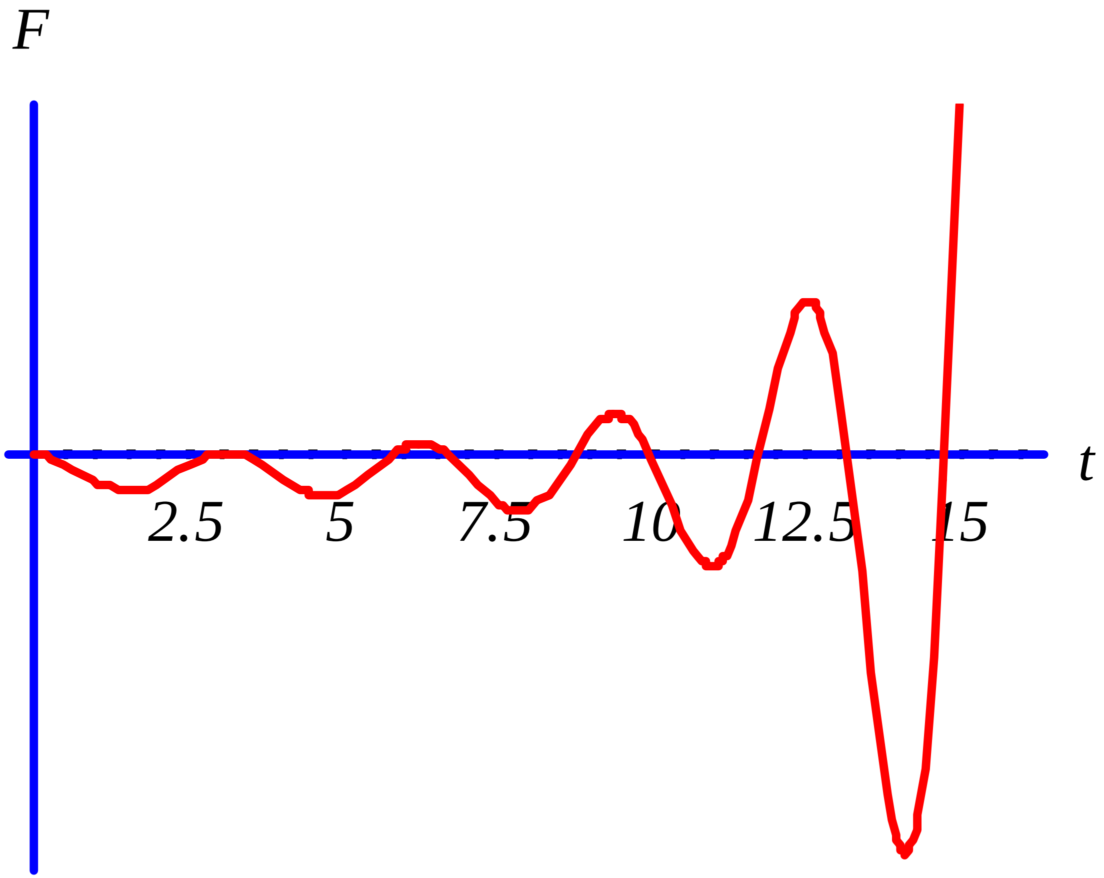
$t < 2d/c$ *(before the backreaction time)*



The force vanishes at $t = 0$.

At successive times, during the dynamical self-dressing of the atom, the force oscillates from attractive to repulsive (even for an initially ground-state atom).

$t < 2d/c$

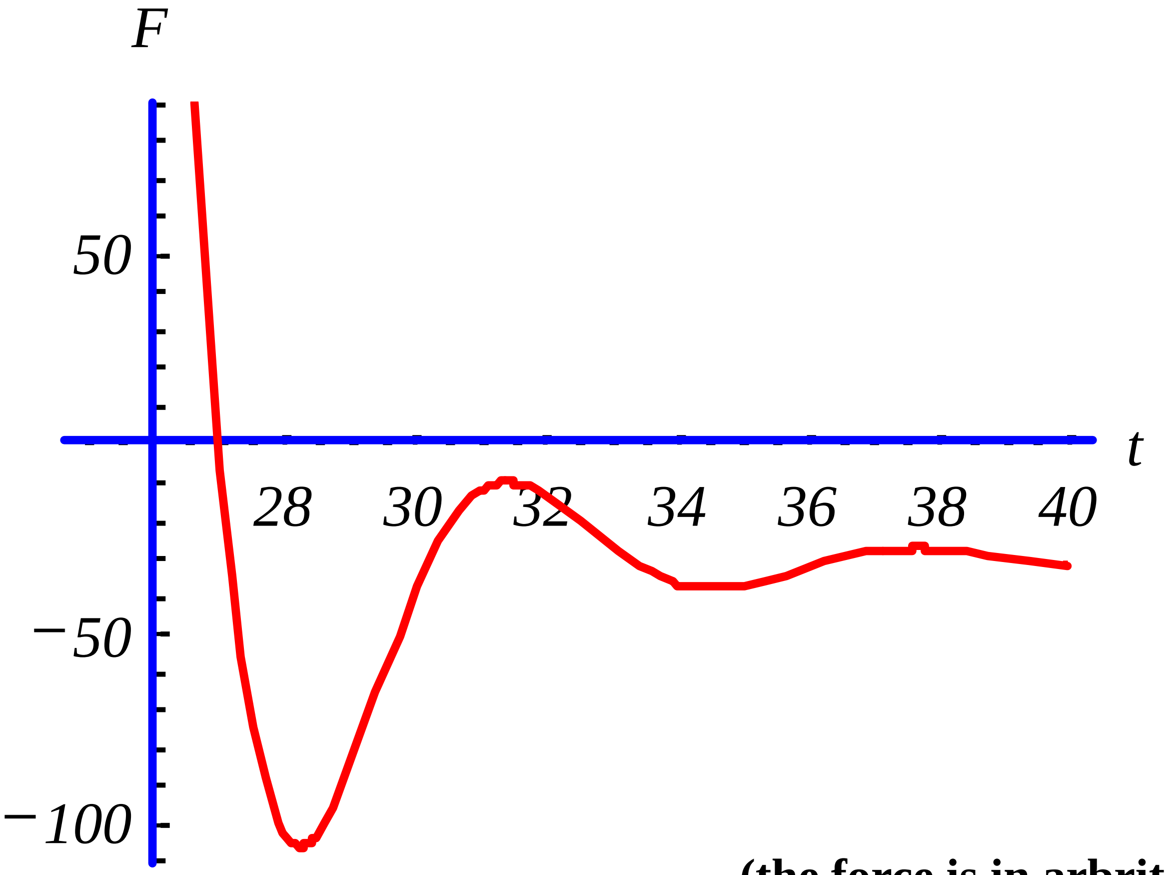


$c = 1, k_0 = 2, d = 10$

the atomic transition frequency is twice than in the previous case

$t > 2d/c$ (after the backreaction time)

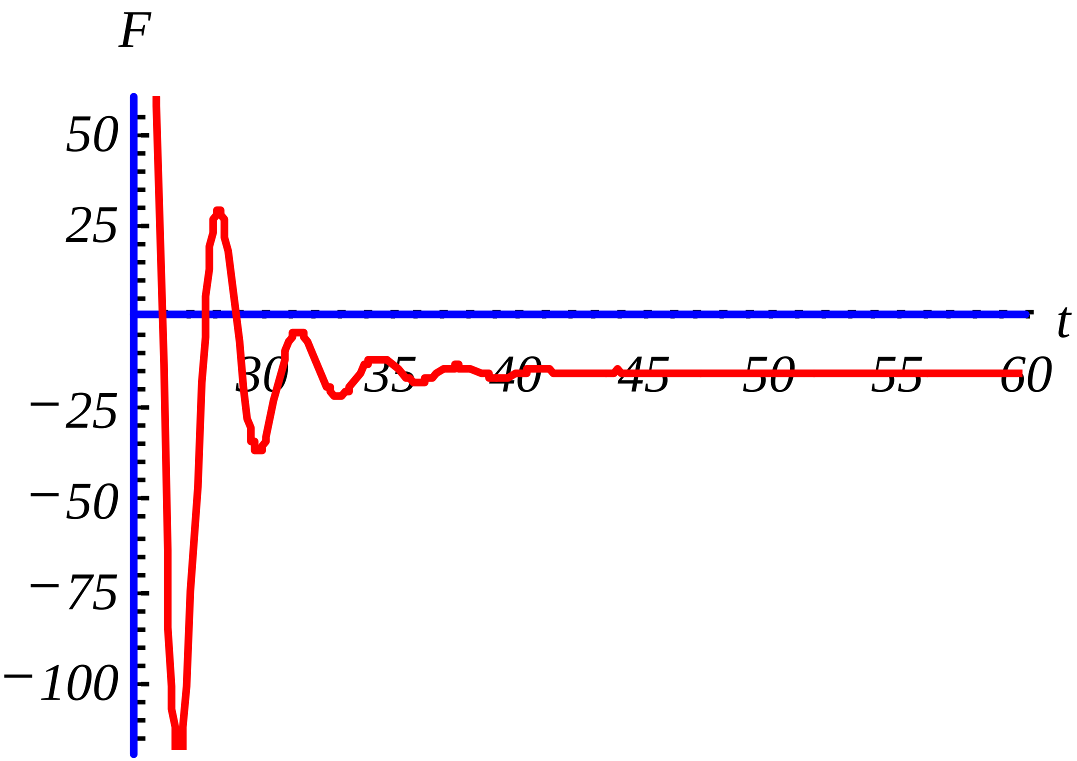
($c = 1$, $k_0 = 1$, $d = 10$)



(the force is in arbitrary units)

$t > 2d/c$

(c = 1, k_0 = 2, d = 10)



(the force is in arbitrary units)

For large times, the force settles to a negative value (attractive force), which coincides with that obtained in the stationary situation (at large times, the atom is *fully dressed*).

R. Vasile, R. Passante, Phys. Rev. A **78**, 032108 (2008)

Dynamical Casimir-Polder force between two atoms in front of a perfectly conducting plate

Observation point
(atom B)

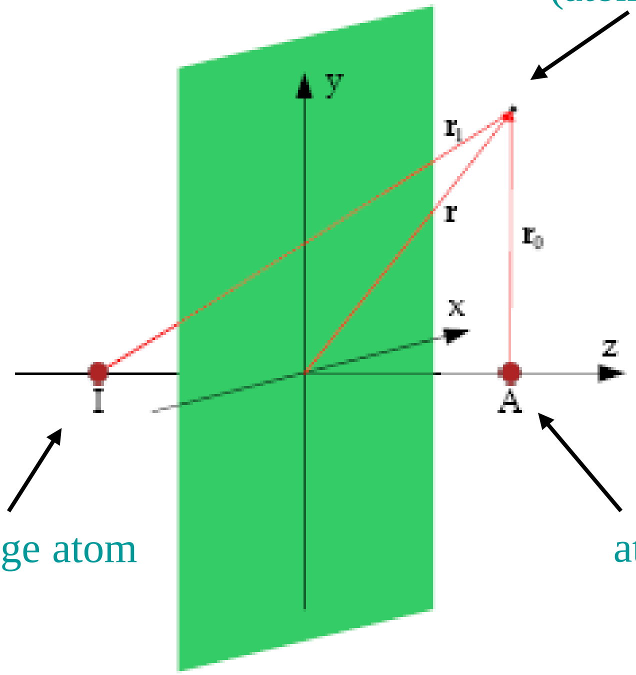


image atom

atom A

“Source” atom A, initially in its bare ground state, generates time-dependent field fluctuations in **r** during its dynamical self-dressing, which propagate with velocity *c* :
“direct” contribution plus “reflected” contribution.

Electric and magnetic interaction energies between the atom A and a second atom B placed in $\mathbf{r}_B$ (in the presence of the plate)

electric polarizability of atom B

$$\Delta E_{AB}^{E} = -\frac{1}{2}\alpha_B^E \left\langle E^2\left(\mathbf{r}_B, t\right)\right\rangle_A$$

magnetic polarizability of atom B

$$\Delta E_{AB}^{M} = -\frac{1}{2}\alpha_B^M \left\langle B^2\left(\mathbf{r}_B, t\right)\right\rangle_A$$

(three contributions to the interaction energy: atom-atom interaction, atom-image interaction, mixed contribution)

Ruggero Vasile, Riccardo Messina, Roberto Passante, Phys. Rev. A **79**, 062106 (2009)

Dynamical atom-wall Casimir-Polder energy for an initially partially dressed state

Possible scheme for the generation of the partially dressed state, i.e. a nonequilibrium configuration of the atom-field system, by a sudden external action on the atom (contrarily to the bare state, the partially dressed state can be generated in the laboratory)

Hamiltonian of a two-level atom interacting with the e.m. radiation field (for *t* < *0*)

$$H = H_{field} + \hbar\omega_0 S_z - \mu\bullet\mathbf{E}(\mathbf{r}_A)$$

atomic transition frequency

position of the atom

Fully dressed state at *t = 0* (in the presence of the plate):

mode functions due to the presence of the conducting plate

$$|g\rangle_d = \left|\downarrow_A, \{0_{\mathbf{k}j}\}\right\rangle - i\sqrt{\frac{2\pi c}{\hbar V}}\sum_{\mathbf{k}j}\frac{\sqrt{\omega}\left(\mu\bullet\mathbf{f}_{\mathbf{k}j}(\mathbf{r}_A)\right)}{\omega+\omega_0}\left|1_{\mathbf{k}j},\uparrow_A\right\rangle$$

bare ground state

dressing photon cloud

t = 0 : sudden change of the atomic transition frequency

$$\omega_0 \to \omega_0' = \omega_0 + \Delta\omega \qquad \Rightarrow H \to H'$$

(example: Stark shift of the atomic levels by rapidly switching on (or switching off) an external electric field acting on the atom)

The state $|g\rangle_d$ is an eigenstate of the "old" Hamiltonian *H*,

but it is *not* an eigenstate of the "new" Hamiltonian *H'*: for t > 0 it is a "partially dressed" state → time evolution

***Non-adiabatic assumption:* immediately after the switching on/off of the external electric field, the state of the system remains unchanged (i.e., the switching on timescale must be smaller than ω_0^{-1}, otherwise the atom would adiabatically follow the change)**

Energy density of the field emitted by an atom in free space starting from a partially dressed state in the far zone $(\hbar = 1)$

$$\frac{1}{8\pi}\left\langle E^2\left(\mathbf{r},t\right)\right\rangle - \frac{1}{8\pi}\left\langle E^2\left(\mathbf{r},t<0\right)\right\rangle =$$

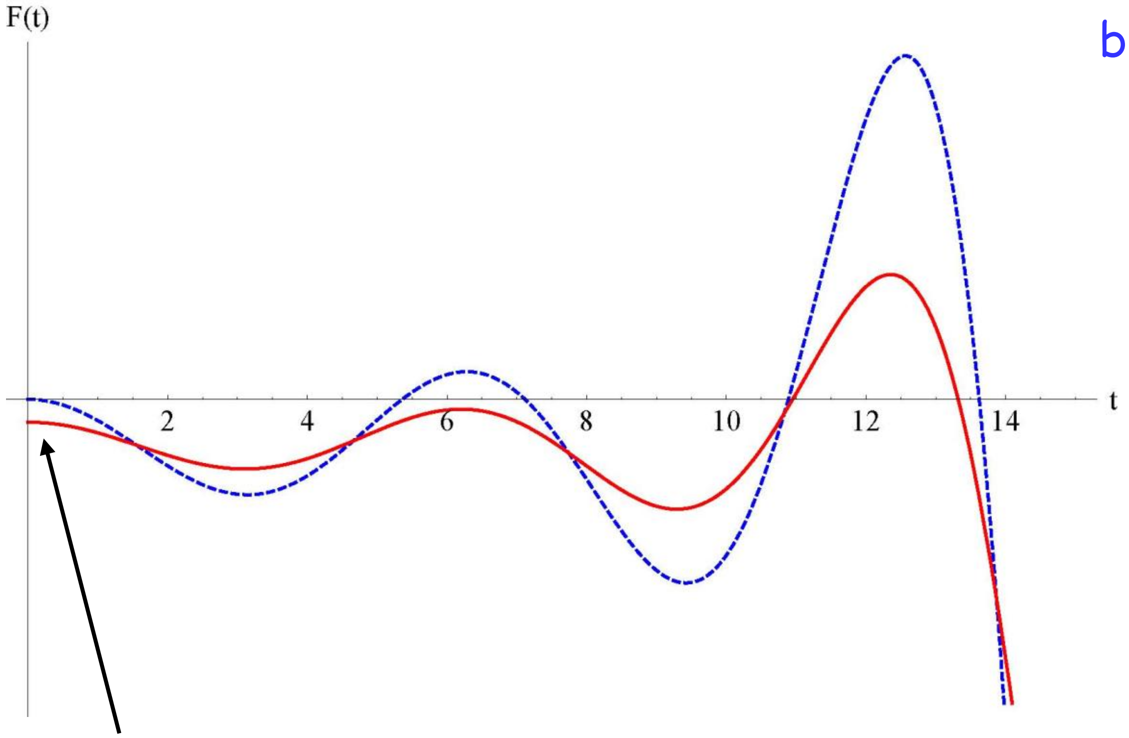
$$= -\Delta\omega_0 \frac{cd^2}{24\pi^2\omega_0^2}\{\frac{13}{2r^7}\left[1-\Theta\left(r-ct\right)\right] + \frac{13}{2r^6}\delta\left(r-ct\right) - \frac{5}{2r^5}\delta'\left(r-ct\right) +$$

$$+\frac{1}{3r^4}\delta''\left(r-ct\right) + \frac{1}{30r^2}\delta^{(iv)}\left(r-ct\right)\}$$

R. Passante, F. Persico, Phys. Lett. A 312, 319 (2003)

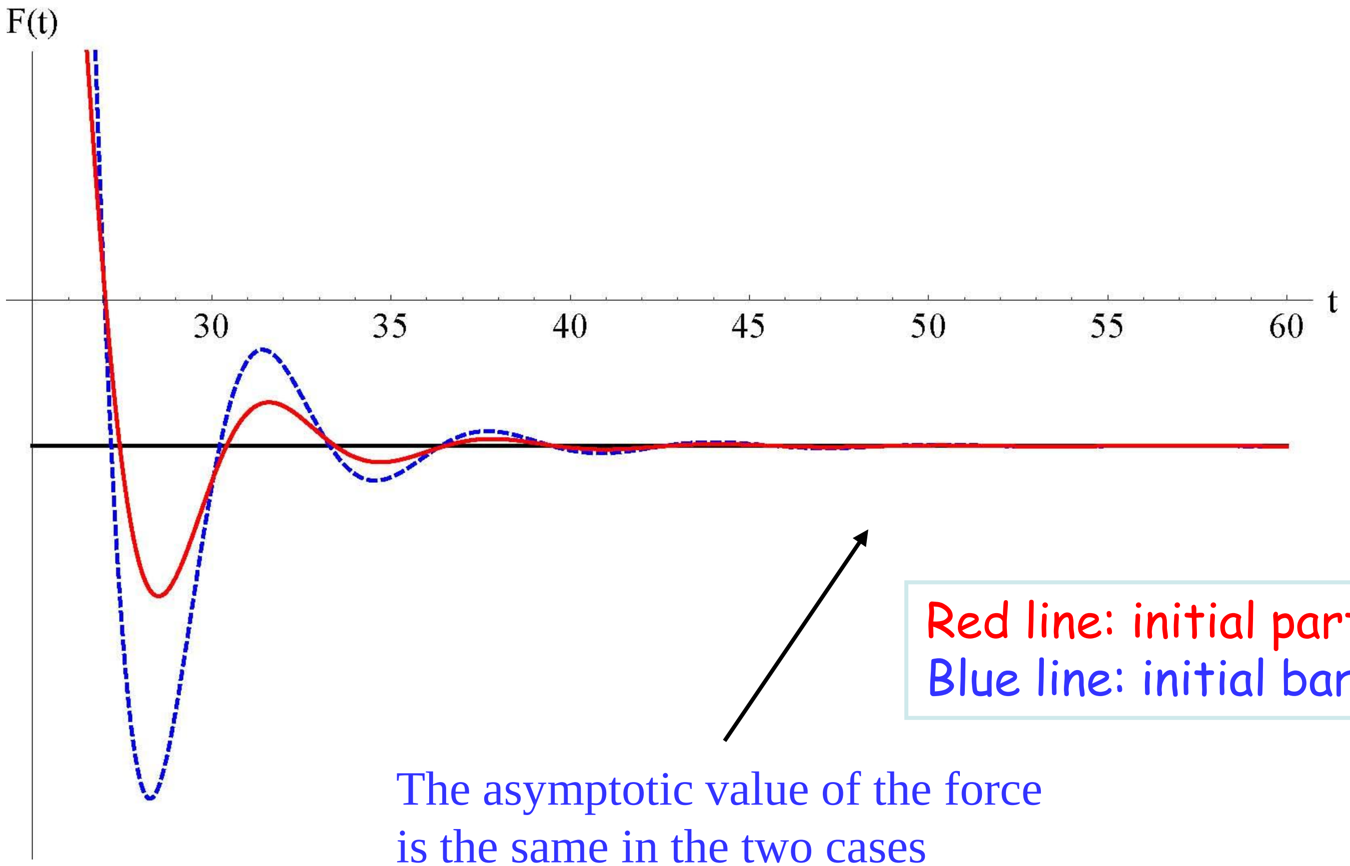
Dynamical Casimir-Polder force for the initial partially dressed state

red line: initial partially dressed state
blue line: initial bare state



$t < 2d \quad (c=1)$
$d = 10$
$\omega_0 = 1$
$\omega_0' = 2$

in the case of the partially dressed state, the force is ***not*** zero at $t = 0$



t > 2d (c=1)
d = 10
$\omega_0 = 1$
$\omega_0' = 2$

Red line: initial partially dressed state
Blue line: initial bare state

The asymptotic value of the force is the same in the two cases

For t → ∞, local field configurations are the same in the three cases (dynamical cases with bare and partially dressed initial state, static case); they only differ for the field at infinite distance from the atom.

Riccardo Messina, Roberto Passante, Ruggero Vasile, Phys. Rev. A **82**, 062501 (2010)

Constraints for a possible experimental verification of the dynamical effect and of the force oscillations

timescale of the external action on the atom

- **Validity of the non-adiabatic hypothesis:** $\Delta t << \omega_0^{-1}$

- **Realistic value of $\Delta\omega_0$ (that is of the "external electric field")**

- **Intensity of the dynamical force and of its oscillations**

- **Very fast measurement of the force**

Rydberg atom in a metastable state?

Conclusions

Dynamical Casimir-Polder force between an atom and a conducting wall, starting from a nonequilibrium configuration of the system (bare ground or excited state).

Emission of radiation during the atomic dynamical dressing.

Dynamical Casimir-Polder force for an initially partially dressed state.

Oscillation of the sign of the dynamical Casimir-Polder force.

Possible experimental verification in the case of an initial partially dressed state.