Tetrahedral Symmetry in Nuclei: Theory and Experimental Criteria

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The Most Fundamental Issue In Particle Physics
are fundamental objects: quarks, leptons, gauge-bosons

* * *

The Most Fundamental Issue in Nuclear Physics
is nuclear existence: nuclear masses and thus nuclear stability

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theoretical ideas and examples of an experimental evidence
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Superheavy Nuclei with new [tetrahedral] SH magic numbers!

Nuclei Known Today

New Islands of Stability
[special astrophysical consequences]

Proton Number

Neutron Number
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Tetrahedron, Cube, Octahedron, Icosahedron, Dodecahedron
The symbol of ’beauty in symmetry’ are five Platonic Figures.
There exist only five regular convex (=platonic) polyhedra: 
**tetrahedron, cube, octahedron, icosahedron & dodecahedron**

As it seems, neolithic people from Scotland have developed the five Platonic solids about 1000-3000 years before Plato (stone models in Ashmolean Museum, Oxford) ... in religious context.

In what follows we stick to the aspect of beauty and nuclear reality - similarity to any other context will be purely accidental.
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In what follows we stick to the aspect of beauty and nuclear reality - similarity to any other context will be purely accidental.
Non-Trivial Discrete Symmetries in Nuclei: $T_d$

Let us recall one of the magic forms introduced long time by Plato. The implied symmetry leads to the *tetrahedral group* denoted $T_d$.

A tetrahedron has four equal walls. Its shape is invariant with respect to 24 symmetry elements. Tetrahedron is *not* invariant with respect to the inversion. Of course nuclei cannot be represented by a sharp-edge pyramid, but rather in a form of a regular spherical harmonic expansion:

$$R(\vartheta, \varphi) = R_0 c(\{\alpha\})[1 + \sum_{\lambda}^{\lambda_{\text{max}}} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda,\mu} Y_{\lambda,\mu}(\vartheta, \varphi)]$$
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... but rather in a form of a regular spherical harmonic expansion:

$$R(\vartheta, \varphi) = R_0 \left\{ 1 + \alpha_{3+2} (Y_{3+2} + Y_{3-2}) + \alpha_{72} \left[ (Y_{7+2} + Y_{7-2}) - \sqrt{\frac{11}{13}} (Y_{7+6} + Y_{7-6}) \right] \right\}$$

one parameter 3rd order

one parameter 7th order
Introducing Nuclear Octahedral Symmetry

Let us recall one of the magic forms introduced long time by Plato. The implied symmetry leads to the *octahedral group* denoted $O_h$.

An octahedron has 8 equal walls. Its shape is invariant with respect to 48 symmetry elements that include inversion. However, the nuclear surface cannot be represented in the form of a diamond → → → → → → → →

... but rather in a form of a regular spherical harmonic expansion:

$$\mathcal{R}(\vartheta, \varphi) = R_0 c(\{\alpha\})[1 + \sum_{\lambda=0}^{\lambda_{\text{max}}} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda,\mu} Y_{\lambda,\mu}(\vartheta, \varphi)]$$
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An octahedron has 8 equal walls. Its shape is invariant with respect to 48 symmetry elements that include inversion. However, the nuclear surface cannot be represented in the form of a diamond \(\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow\)

\[ \mathcal{R}(\vartheta, \varphi) = R_0 \left\{ 1 + \alpha_{40} \left[ Y_{40} + \sqrt{\frac{5}{14}} (Y_{4+4} + Y_{4-4}) \right] + \alpha_{60} \left[ Y_{60} - \sqrt{\frac{7}{2}} (Y_{6+4} + Y_{6-4}) \right] \right\} \]

... but rather in a form of a regular spherical harmonic expansion:
Consider a nuclear surface with tetrahedral deformation ...

This is what we call in jargon: ‘nuclear pyramid’ = tetrahedron
... and another surface with octahedral deformation...

This is what we call in jargon: ‘nuclear diamond’ = octahedron
... or even better, compare them directly ...

You most likely\(^1\) clearly see the difference between the two shapes: the pyramid [left] and the diamond [right]

\(^1\) As one student said: "Pyramids are in Egypt" and "Diamonds Are a Girl's Best Friend"; ... but she studied bio-physics...
Surprise-Surprise! Difficult to believe!
Superposing the tetrahedral- and octahedral-symmetry surfaces gives us a new surface but again of tetrahedral symmetry: new richness.
Although by far not intuitive - but it is strict mathematical truth!

It is a manifestation of the theorem saying that:

"Tetrahedral group is a **sub-group** of the octahedral one"
A few words before starting:

**Why Should We Be Interested in All That?**

*First class of reasons:*

- Within nuclear mean-field theory the tetrahedral and octahedral symmetries imply the highest degeneracies of the nucleonic levels → big single-particle gaps and high nuclear stability
- Most economic search for nuclear stability as compared to the old spherical harmonic expansions [only very few degrees of freedom]
A few words before starting:

**Why Should We Be Interested in All That?**

*Second class of reasons:*

- Tetrahedral and octahedral symmetries are only a forefront of group-theory based families of new nuclear symmetries
- The approach uses two most powerful tools: group theory and the nuclear mean-field theory → the best what we have at the beginning of the XXI\textsuperscript{st} century
- All other so far known results, criteria of nuclear stability etc. are just a particular case of the new formulation
Part I

Outline of the New Theory of Nuclear Stability
Consider a typical outcome of the Mean-Field calculation: the shell structures and the total energies.

- Presence of sufficiently strong gaps correlates with local minima of the total nuclear energy.

- The ‘Deformation Parameter’ axis represents several deformations of the mean field, e.g., \( Q_{\lambda\mu} \), \( \alpha_{\lambda\mu} \), the usual approach in constrained mean-field calculations.
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The ‘Deformation Parameter’ axis represents several deformations of the mean field e.g. \( \{ Q_{\lambda\mu} \} \), \( \{ \alpha_{\lambda\mu} \} \), the usual approach in constrained mean-field calculations.
Given Hamiltonian $H$ and a group: $G = \{O_1, O_2, \ldots O_f\}$

Assume that $G$ is a symmetry group of $H$ i.e.

$$[H, O_k] = 0 \quad \text{with} \quad k = 1, 2, \ldots f$$

Let irreducible representations of $G$ be $\{R_1, R_2, \ldots R_r\}$

Let their dimensions be $\{d_1, d_2, \ldots d_r\}$, respectively

Then the eigenvalues $\{\varepsilon_\nu\}$ of the problem

$$H \psi_\nu = \varepsilon_\nu \psi_\nu$$

appear in multiplets $d_1$-fold, $d_2$-fold ... degenerate
Symmetries, Representations and Degeneracies

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Symmetries and Gaps in Nuclear Context: Schematic

Schematic illustration: Levels of 6 irreps and average spacings/gaps

Roughly: The average level spacings within an irrep increase by a factor of 6. The total spectrum may present big unprecedented gaps.
Symmetries and Gaps in Nuclear Context: Schematic

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Roughly: The average level spacings **within an irrep** increase by a factor of 6. The total spectrum may present big unprecedented gaps.
Symmetries and Gaps in Nuclear Context: Summary

To increase the chances of having big gaps in the spectra we either look for point groups with high dimension irreps or with many irreps.

This implies that we need to verify in the group-theory literature: Which groups have many irreps? ... and/or high dimension irreps?

In other words:

We suggest replacing the multipole expansion in favour of a selection of point groups when studying nuclear stability.

It turns out that the above ideas are well confirmed by calculations.
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Double group $O_h^D$ has four 2-dimensional and two 4-dimensional irreducible representations $\rightarrow$ six distinct families of levels

**Figure**: Full lines correspond to 4-dimensional irreducible representations - they are marked with double Nilsson labels. Observe huge gap at $Z=70$.  

**Jerzy DUDEK, University of Strasbourg, France**

**Tetrahedral Symmetry in Nuclei: Theory, Experimental Criteria**
Example: Octahedral Symmetry - Neutron Spectra

Double group $O^D_h$ has four 2-dimensional and two 4-dimensional irreducible representations $\rightarrow$ six distinct families of levels

Figure: Full lines correspond to 4-dimensional irreducible representations - they are marked with double Nilsson labels. Observe huge gap at $N=114$. 
Part II

Focus on Nuclear Tetrahedral Symmetry
First Goal: Obtain Tetrahedral Magic Numbers ...

1. After inspecting many single-particle diagrams in function of tetrahedral deformation we read-out all the magic numbers

2. The tetrahedral symmetric nuclei are predicted to be particularly stable around magic closures:

\[ \{Z_t, N_t\} = \{32, 40, 56, 64, 70, 90, 136\} \]

... and more precisely around the following nuclei:

\[
\begin{align*}
\text{ }_{32}^{64}\text{Ge}, & \quad \text{ }_{32}^{72}\text{Ge}, & \quad \text{ }_{32}^{88}\text{Ge}, & \quad \text{ }_{40}^{80}\text{Zr}, & \quad \text{ }_{40}^{110}\text{Zr}, & \quad \text{ }_{56}^{112}\text{Ba}, \\
\text{ }_{56}^{126}\text{Ba}, & \quad \text{ }_{56}^{146}\text{Ba}, & \quad \text{ }_{64}^{134}\text{Gd}, & \quad \text{ }_{64}^{154}\text{Gd}, & \quad \text{ }_{70}^{160}\text{Yb}, & \quad \text{ }_{90}^{226}\text{Th}.
\end{align*}
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3. ... and more precisely around the following nuclei:

   \[
   ^{64}_{32}\text{Ge}_{32}, \quad ^{72}_{32}\text{Ge}_{40}, \quad ^{88}_{32}\text{Ge}_{56}, \quad ^{80}_{40}\text{Zr}_{40}, \quad ^{110}_{40}\text{Zr}_{70}, \quad ^{112}_{56}\text{Ba}_{56},
   \]

   \[
   ^{126}_{56}\text{Ba}_{70}, \quad ^{146}_{56}\text{Ba}_{90}, \quad ^{134}_{64}\text{Gd}_{70}, \quad ^{154}_{64}\text{Gd}_{90}, \quad ^{160}_{70}\text{Yb}_{90}, \quad ^{226}_{90}\text{Th}_{136}
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\[ \frac{64}{32} Ge_{32}, \frac{72}{32} Ge_{40}, \frac{88}{32} Ge_{56}, \frac{80}{40} Zr_{40}, \frac{110}{40} Zr_{70}, \frac{112}{56} Ba_{56}, \]

\[ \frac{126}{56} Ba_{70}, \frac{146}{56} Ba_{90}, \frac{134}{64} Gd_{70}, \frac{154}{64} Gd_{90}, \frac{160}{70} Yb_{90}, \frac{226}{90} Th_{136} \]
Tetrahedral Stability; Tetrahedral Magic Numbers

Tetrahedral Symmetry Induced Magic Numbers

Neutron Number

Proton Number

N=40
N=56
N=70
N=90
N=112
N=136
N=178
Z=40
Z=56
Z=64
Z=70
Z=90
Z=112
Z=118
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Focus on the Nuclear Tetrahedral Symmetry

Tetrahedral Stability; Tetrahedral Magic Numbers

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Tetrahedral Symmetry in Nuclei: Theory, Experimental Criteria
Experimental Hints: Where Are Tetrahedra?

- An original competition between quadrupole and tetrahedral shapes
- Observe that tetrahedral minima may lie very high
- Finding optimum experimental conditions is a matter of a compromise

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The phenomenon discussed is a low-spin and relatively high-energy effect.
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Do We Know How the Axial Nuclei Rotate?

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Spin-Orientation Probability

I=10, n=1; $E_{\text{rot}}=1.00000000000 \ (A1)$

Z=40, N=40, $J=24.5, J_x=24.5, J_y=24.5, J_z=18.8$; No-higher-defs.
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Tetrahedral Symmetry in Nuclei: Theory, Experimental Criteria
**Spin-orientation probability at the constant spin $I$ of an axially symmetric nucleus - observe the evolution with nuclear excitation at fixed value $I=14$**
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Spin-orientation probability at the constant spin I of an axially symmetric nucleus - observe the evolution with nuclear excitation at fixed value I=14
What did we learn from these pictures?

- Nearly trivial case of the yrast states goes along with our intuition
- Anything else is anti-intuitive: it can be seen as a surprise!
- However: these pictures carry a very neat geometrical information
- To our knowledge nobody asks this type of questions in nuclear physics - although there are entire books discussing the analogous problems in molecular physics!
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• First message is this: we know the wave functions, we can and we do calculate the transition probabilities that give necessary and sufficient experimental criteria for the presence of the symmetries
• These pictures help to understand and interpret geometrically the theoretical predictions for electromagnetic transition probabilities
• ... but this is a looong topic for yet another, separate story!
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- These pictures help to understand and interpret geometrically the theoretical predictions for electromagnetic transition probabilities
- ... but this is a looong topic for yet another, separate story!
This stable nucleus is predicted tetrahedral-unstable in the ground-state - and yet will combine the signs of sphericity and tetrahedrality.
Survey of Doubly-Magic Tetrahedral Nuclei

This nucleus is predicted to manifest tetrahedral minima in competition with the quadrupole ground-state minimum ...
Survey of Doubly-Magic Tetrahedral Nuclei

... and in competition with the pear-shape ‘old’ octupole minimum !!!
Multipole Moments as Functionals of the Density

- Nuclear surface $\Sigma$ is defined in terms of multipole deformations:
  \[ \Sigma : \quad R(\vartheta, \varphi) = R_0 \left[ 1 + \sum_\lambda \sum_\mu \alpha_{\lambda\mu} Y_{\lambda\mu}(\vartheta, \varphi) \right] \]

- Given uniform density $\rho_\Sigma(\vec{r})$ defined using the surface $\Sigma$:
  \[ \rho_\Sigma(\vec{r}) = \begin{cases} 
  \rho_0 : & \vec{r} \in \Sigma \\
  0 : & \vec{r} \not\in \Sigma 
\end{cases} \]

- Express the multipole moments as usual by:
  \[ Q_{\lambda\mu} = \int \rho_\Sigma(\vec{r}) r^\lambda Y_{\lambda\mu} \, d^3\vec{r} \]

- We will calculate the quadrupole moments as functions of $\alpha_{3\mu}$.
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- Express the multipole moments as usual by

$$Q_{\lambda \mu} = \int \rho_\Sigma(\vec{r}) r^\lambda Y_{\lambda \mu} \, d^3\vec{r}$$

- We will calculate the quadrupole moments as functions of $\alpha_{3 \mu}$
For small deformations we use Taylor expansion:

\[ Q_{\lambda \mu}(\alpha) \approx Q_{\lambda \mu}\bigg|_{\alpha=0} + Q'_{\lambda \mu}\bigg|_{\alpha=0} \Delta \alpha + \frac{1}{2} Q''_{\lambda \mu}\bigg|_{\alpha=0} \Delta \alpha \Delta \alpha \]

We set \( \lambda = 2 \), \( \mu = 0 \) and \( \lambda_1 = \lambda_2 = 3 \) and obtain:

\[ \alpha_{30} : \quad Q_{20} = \frac{15}{2 \sqrt{5 \pi}} \cdot \alpha_{30}^2 \cdot \rho_0 R_0^5 \]
\[ \alpha_{31} : \quad Q_{20} = \frac{15}{4 \sqrt{5 \pi}} \cdot \alpha_{3+1} \alpha_{3-1} \cdot \rho_0 R_0^5 \]
\[ \alpha_{32} : \quad Q_{20} = 0 \]
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Conclusion: Among \( \lambda = 3 \) defs. only \( \alpha_{32} \) leads to \( Q_2 \equiv 0 \) !!!
Focus on the Nuclear Tetrahedral Symmetry

Expected Experimental Signs of Tetrahedral Symmetry

Multipole Moments as Functionals of the Density

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Jerzy DUDEK, University of Strasbourg, France

Tetrahedral Symmetry in Nuclei: Theory, Experimental Criteria
Focus on the Nuclear Tetrahedral Symmetry

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Jerzy DUDEK, University of Strasbourg, France

Tetrahedral Symmetry in Nuclei: Theory, Experimental Criteria
We Proceed to Suggest a Physical Interpretation

- When looking for experimental signs of tetrahedral symmetry assume that $Q_2$-moments of tetra-configurations nearly vanish

- Strictly speaking: Exact tetrahedral and octahedral symmetries cause vanishing of quadrupole and dipole moments: the only permitted transitions are octupole - an extreme idea!

- We know that at reduced transition probabilities $B(E1)$ and $B(E3)$ comparable, the E1-decay is $\sim 10^{12}$ more probable!!

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Focus on the Nuclear Tetrahedral Symmetry

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Focus on the Nuclear Tetrahedral Symmetry

Extreme-Symmetry Limit: $Q_2 = 0$ and $Q_1 = 0$

Dipole Moment non-zero

Dipole Moment vanishes

Transitions always present

The only transitions are the octupole ones

Pear-shape

Tetrahedral

Jerzy DUDEK, University of Strasbourg, France

Tetrahedral Symmetry in Nuclei: Theory, Experimental Criteria
Denoting quadrupole deformations $\alpha$ and classical energy $E$ we have:

$$E(\alpha, \dot{\alpha}) = \frac{1}{2} B \dot{\alpha}^2 + \frac{1}{2} C \alpha^2 \quad \rightarrow \quad \hat{H} = \frac{1}{2B} \frac{\partial^2}{\partial \alpha^2} + \frac{1}{2} C \alpha^2,$$

The normalised wave functions are:

$$\varphi_n(\alpha) = \frac{1}{\sqrt{\sqrt{2\pi} n!}} \cdot \frac{1}{A} \cdot e^{-\alpha^2/2\sigma^2} H_n(\alpha/\sigma), \quad \sigma \overset{df.}{=} \sqrt{2A},$$

where

$$A \overset{df.}{=} [\langle \varphi_{n=0} | \alpha^2 | \varphi_{n=0} \rangle]^{1/2} = [\hbar^2/(4BC)]^{1/4}$$

We find an effective quadrupole deformation different from zero:

Most probable deformation:

$$\langle |\alpha| \rangle \sim \int |\alpha| \varphi_n^2(\alpha) \, d\alpha \neq 0$$
Focus on the Nuclear Tetrahedral Symmetry

Quadrupole Polariisation through Zero-Point Motion

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Focus on the Nuclear Tetrahedral Symmetry

We Have the Smoking-Gun Signatures Almost There

Valence particles cause a certain quadrupole polarisation

Additional polarisation caused by Coriolis spin alignments

Spin-alignment will cause additional quadrupole polarisation

Additional polarisation

Spin alignments

Total Spin

Jerzy DUDEK, University of Strasbourg, France
According to a simplified way of thinking, when all deformations tend to zero ($\alpha \lambda \mu \rightarrow 0$) then $Q_2 \rightarrow 0$ and $Q_1 \rightarrow 0$ and we are confronted with an ill-defined mathematical problem.

$$\lim_{\alpha \rightarrow 0} \frac{B(E2)}{B(E1)} = ??? \quad (\text{undefined symbol} \frac{0}{0}) !!!$$

However, because of the residual polarisations in terms of quadrupole deformation and of induced dipole moments at the band-heads we have

$$\lim_{\alpha \rightarrow 0} \frac{B(E2)}{B(E1)} = \frac{B_{\text{res}}(E2)}{B_{\text{res}}(E1)} \equiv B_0 \neq 0$$
We Have the Smoking-Gun Signatures Almost There ...

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We Have the Smoking-Gun Signatures Almost There ...

In other words, we expect a spin dependence:

\[ \frac{B(E2)}{B(E1)} \sim B_0 + B_1 \cdot I \]

**Conclusion:** Tetrahedral symmetry must always be accompanied by static or dynamic quadrupole deformations at \( \alpha_{20} \neq 0 \) and \( \alpha_{22} \neq 0 \).
We Have the Smoking-Gun Signatures Almost There ...

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### Possible El-Magnetic Signs of Tetrahedral Symmetry

**Table: Experimental ratios $B(E2)_{in}/B(E1)_{out} \times 10^6$**

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Above: Branching ratios related to the negative parity bands are interpreted as tetrahedral, interband transitions to g.s.band.
**Possible El-Magnetic Signs of Tetrahedral Symmetry**

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Part III

We can now formulate further experimental criteria!

**The Story of the ‘Smoking Guns’**

- Tetrahedral nuclei are deformed → they produce collective rotation
- The lowest order $T_d$–symmetry is $Y_{3±2} →$ negative parity bands
- At the exact symmetry limit $Q_2$ moments must vanish! Therefore:
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We suggest looking for the collective negative parity bands without ‘rotational’ (E2) transitions. The question – Where?
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New Theory: Verification, Relation to Experiment

Constructing a Series of Theoretical Predictions

Experimental Facts: Comparison with Experiment

Where to Look for Experimental Evidence: Summary

The Strongest Tetrahedral Islands Predicted by Theory

In the Actinide region
most of the so-called octupole bands
have never seen their $E_2$ transitions
in experiment  [detailed discussion]
The Strongest Tetrahedral Islands Predicted by Theory

In the Rare Earth Region
the Sm, Gd and Dy nuclei \([Z=62,64,66]\)
manifest negative parity bands
without E2 transitions [see details]
Where to Look for Experimental Evidence: Summary

The Strongest Tetrahedral Islands Predicted by Theory

In the Zirconium region, several nuclei manifest largest ever octupole transitions \([B(E3) \sim (20-60) \text{W.u.}]\).

Jerzy DUDEK, University of Strasbourg, France
Tetrahedral Symmetry in Nuclei: Theory, Experimental Criteria
Example: This Is Not Our Effect [Pear-Shape]

E(fyu)+Shell[e]+Correlation[PNP]

E[MeV]  11.50  11.00  10.50  10.00  9.50  9.00  8.50  8.00  7.50  7.00  6.50  6.00  5.50  5.00  4.50  4.00  3.50  3.00  2.50  2.00  1.50  1.00  0.50  0.00

Deformation $\alpha_{20}$

Deformation $\alpha_{30}$

$^{222}_{90}$Th$_{132}$  Emin=-7.58, Eo=-3.07

Jerzy DUDEK, University of Strasbourg, France

Tetrahedral Symmetry in Nuclei: Theory, Experimental Criteria
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\[ B(E2)_{in}/B(E1)_{out} \times 10^6 e\,Fm \]

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Tetrahedral Symmetry in Nuclei: Theory, Experimental Criteria

Jerzy DUDEK, University of Strasbourg, France
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Conclusion: Tetrahedral bands in Rare Earth nuclei behave very differently as compared e.g. to 'classical' octupole $^{222}\text{Th}$ band!

Jerzy DUDEK, University of Strasbourg, France
## Possible El-Magnetic Signs of Tetrahedral Symmetry

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<td>24</td>
<td>-</td>
<td>-</td>
<td>+0.4</td>
</tr>
<tr>
<td>$^{13-}$</td>
<td>14</td>
<td>7</td>
<td>15</td>
<td>18</td>
<td>23</td>
<td>-</td>
<td>17</td>
<td>+0.3</td>
</tr>
<tr>
<td>$^{11-}$</td>
<td>4</td>
<td>15</td>
<td>5</td>
<td>9</td>
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<td>10</td>
<td>11</td>
<td>+0.4</td>
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<tr>
<td>$^{9-}$</td>
<td>4</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td>11</td>
<td>10</td>
<td>+0.4</td>
</tr>
<tr>
<td>$^{7-}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>+0.4</td>
</tr>
</tbody>
</table>

**Conclusion:** Tetrahedral bands in Rare Earth nuclei behave very differently as compared e.g. to 'classical' octupole $^{222}\text{Th}$ band!
Partial Decay of $^{156}$Gd and Vanishing $Q_2$-Moments

From $I^\pi = 9^-$ down - no E2-transitions are observed despite tries

<table>
<thead>
<tr>
<th>Process</th>
<th>Refs</th>
<th>Last Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{156}$Eu: $\beta$-decay</td>
<td>43</td>
<td>1995</td>
</tr>
<tr>
<td>$^{156}$Tb: ec-decay</td>
<td>38</td>
<td>1995</td>
</tr>
<tr>
<td>$^{150}$Nd: ($^{13}$C,A-3n-$\gamma$)</td>
<td>2</td>
<td>2001</td>
</tr>
<tr>
<td>$^{154}$Sm: (A,2n-$\gamma$)</td>
<td>11</td>
<td>2001</td>
</tr>
<tr>
<td>$^{154}$Gd: (t,p)</td>
<td>1</td>
<td>1989</td>
</tr>
<tr>
<td>$^{155}$Gd: (n,$\gamma$)</td>
<td>40</td>
<td>2000</td>
</tr>
<tr>
<td>$^{155}$Gd: (d,p)</td>
<td>2</td>
<td>1994</td>
</tr>
<tr>
<td>$^{156}$Gd: ($\gamma$,$\gamma'$),(e,e')</td>
<td>27</td>
<td>2000</td>
</tr>
<tr>
<td>$^{156}$Gd: ($\mu$,$\gamma$)</td>
<td>1</td>
<td>1971</td>
</tr>
<tr>
<td>$^{156}$Gd: (n,n'$\gamma$)</td>
<td>3</td>
<td>1996</td>
</tr>
<tr>
<td>$^{156}$Gd: (p,p'$\gamma$),(d,d')</td>
<td>5</td>
<td>1989</td>
</tr>
<tr>
<td>$^{157}$Gd: (p,d),(3He,A)</td>
<td>2</td>
<td>1984</td>
</tr>
<tr>
<td>$^{157}$Gd: (d,t)</td>
<td>1</td>
<td>1993</td>
</tr>
<tr>
<td>$^{158}$Gd: (p,t)</td>
<td>8</td>
<td>1982</td>
</tr>
<tr>
<td>Coulomb excitation</td>
<td>25</td>
<td>1993</td>
</tr>
</tbody>
</table>

According to C. W. Reich, Nucl. Data Sheets 99 753 (2003) a few dozens among those refs have been used to deduce the level scheme on the right...
Tetrahedral/Octahedral Shapes Have No $Q_2$-Moments

At the exact tetrahedral symmetry the quadrupole moments vanish

Equilibrium shape $t_1 = 0.15$

...but, $E2$-intensities are expected to grow with spin (Coriolis polarisation)
Comparison between three typical hypotheses:
1. Tetrahedral and Octahedral $\leftrightarrow$ (defs. from microscopic calculations);
2. Tetrahedral and Octahedral + Zero-Point Motion $(\alpha_{20}^{\text{pol}} = 0.07)$;
3. Prolate ground-state deformation with $\alpha_{20} \approx 0.25$

![Graph showing alignment of simulations with Gd-156 experimental data.](image-url)
1. Our first-time evidence for the non-stretched E2-transitions from the rotational even-spin negative-parity band to the tetrahedral-suspect band.

2. These transitions support the interpretation of tetrahedral sequence as a real rotational band, fed and depopulated through $\Delta I = 1$ transitions!
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What Did We Learn So Far from Jyvaskyla Experiment

First Conclusions:

- We believe that the odd-spin negative-parity band in $^{156}$Gd must not be an octupole-vibrational band; octupole vibrations take place around a quadrupole-deformed minimum;
- It cannot be an octupole vibration about the spherical shape either because spherical nuclei have irregular excitation spectra;
- We believe it should be interpreted as tetrahedral band, therefore $K \neq 0$ and the associated bands must undergo $K$-mixing;
- This band is very special: it is fed and depopulated by $\Delta I = 1$ transitions - the $\Delta I = 2$ stretched $E2$-transitions are absent or evidently weak!
- Does the even-spin negative-parity band have extremely low $E1$’s? ... or suddenly strong $E2$’s? Or neither of the two - the only mechanism being changing of the ratio?
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Evidence for Vanishing E2 Transitions in Actinides

The E2 transitions not seen in $^{230-234}\text{U}$, while seen in $^{236}\text{U}$; the experimental conditions ($\gamma$ and $\text{ec}$) are the same or comparable.
Observe the 'tetrahedral' band patterns (vanishing E2-transitions) in $^{230-232}\text{U}$ in both the negative and positive parities!

Jerzy DUDEK, University of Strasbourg, France
### Tetrahedral-Symmetry Candidates in the Actinides

#### Experiment: Three types of situations correspond to three colours:
- **[red]**-tetrahedral,
- **[brown]**-octupole,
- **[green]**-both.

<table>
<thead>
<tr>
<th>N</th>
<th>130</th>
<th>132</th>
<th>134</th>
<th>136</th>
<th>138</th>
<th>140</th>
<th>142</th>
<th>144</th>
<th>146</th>
<th>148</th>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>244</td>
<td>246?</td>
<td>248</td>
</tr>
<tr>
<td>Pu</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Ra</td>
<td>218</td>
<td>220</td>
<td>222</td>
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<td>230</td>
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<tr>
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<td>216</td>
<td>218</td>
<td>220</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Nearly half of the experimental data on the Actinidae nuclei do not manifest the E2-transitions in the negative-parity bands!

Jerzy DUDEK, University of Strasbourg, France
Electromagnetic Transitions in the Actinide vs. RE

Table: Experimental ratios $B(E2)/B(E1) \times 10^6 [fm^2]$, for intra-band $E2$ transitions vs. inter-band $E1$ transitions. Meaning of symbols: “−” - state has not been observed; “?” - intensities to calculate the branching ratios not available; “(?)” - known information insufficient to obtain error bars.

<table>
<thead>
<tr>
<th>State</th>
<th>$^{220}$Th</th>
<th>$^{222}$Th</th>
<th>$^{224}$Th</th>
<th>$^{226}$Th</th>
<th>$^{228}$Th</th>
<th>$^{152}$Gd</th>
<th>$^{156}$Gd</th>
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</thead>
<tbody>
<tr>
<td>$^{21-}$</td>
<td>no E1</td>
<td>0.2(?)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{19-}$</td>
<td>no E1</td>
<td>0.3(?)</td>
<td>-</td>
<td>2.0(5)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$^{17-}$</td>
<td>no E1</td>
<td>0.4(2)</td>
<td>0.3(1)</td>
<td>2.3(4)</td>
<td>no E1</td>
<td>no E1</td>
<td>50(10)</td>
</tr>
<tr>
<td>$^{15-}$</td>
<td>0.9(2)</td>
<td>0.4(2)</td>
<td>0.4(1)</td>
<td>2.0(4)</td>
<td>no E1</td>
<td>no E1</td>
<td>16(3)</td>
</tr>
<tr>
<td>$^{13-}$</td>
<td>0.2(1)</td>
<td>0.3(2)</td>
<td>0.5(1)</td>
<td>?</td>
<td>13(1)</td>
<td>14(?)</td>
<td>7(2)</td>
</tr>
<tr>
<td>$^{11-}$</td>
<td>0.5(1)</td>
<td>0.4(2)</td>
<td>0.4(1)</td>
<td>2.0(2)</td>
<td>no E2</td>
<td>4(?)</td>
<td>15(7)</td>
</tr>
<tr>
<td>$^{9-}$</td>
<td>0.4(1)</td>
<td>0.4(2)</td>
<td>?</td>
<td>?</td>
<td>no E2</td>
<td>no E2</td>
<td>no E2</td>
</tr>
<tr>
<td>$^{7-}$</td>
<td>0.4(1)</td>
<td>0.4(3)</td>
<td>?</td>
<td>?</td>
<td>no E2</td>
<td>no E2</td>
<td>no E2</td>
</tr>
</tbody>
</table>
Strong High-Rank Symmetries in Super-heavy Elements

- Competition between the tetrahedral and octahedral symmetries

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\[ 266_{108}^{158}\text{Hs} \]

\[ E_{\text{min}} = 5.46, \quad E_0 = 6.82 \]
Strong High-Rank Symmetries in Super-heavy Elements

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Jerzy DUDEK, University of Strasbourg, France

Tetrahedral Symmetry in Nuclei: Theory, Experimental Criteria
Strong Octahedral Effects in Super-heavy Elements

- **Above:** Single particle energy levels for neutrons [left] and protons [right]
- **Observe** neutron gap at $N = 178$ much stronger than the one at $N = 184$
- **Similarly,** the proton octahedral gaps at $Z = 118, 120$ are significantly stronger than the ‘classical’, spherical ones at $Z = 114$ and/or $Z = 126$
Summary

• We presented what we call the *New Theory of Nuclear Stability* based on the nuclear mean-field concepts and group theory.

• On its basis we suggest that the nuclear stability is underlined by spatial symmetries with high number of irreducible representations.

• ... rather than multipole expansion, prolate/oblate shape coexistence etc. - although the latter are a particular case of the former.

• We have illustrated the new theory of stability with two high-rank point-groups: *octahedral*- and its *tetrahedral* sub-group.

• We presented preliminary results of our Jyvaskyla experiment on $^{156}$Gd fully confirming the expectations (E2-transitions weaker than so far reported) and bringing new important results.
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• We have demonstrated through realistic calculations that several point-groups so far never considered in nuclear structure physics lead to very strong shell effects

• In particular: tetrahedral symmetry minima imply the presence of negative parity bands with vanishing E2 transitions

• We have found the presence of those bands in the existing literature in full agreement with our general predictions

• It is suggested that the 'octupole effects', considered so far in the literature, separate into two categories: the ‘traditional’ (‘pears’) and ‘tetrahedral’ (‘pyramids’)
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- We are going to study the relatively low spin states, \((I \leq 20\hbar)\) at relatively high excitations - in Rare Earth and Actinide Nuclei.

- Of high priority are the life-time measurements of the absolute values of quadrupole and dipole moments of the tetrahedral bands; All colleagues possibly interested in joining us will be most welcome.

- In coming months we will perform three experiments along these lines \([^{156}\text{Gd in Legnaro},^{156}\text{Dy at Argonne and}^{154}\text{Gd at iThemba}]\).

- We are extending the new symmetry ideas to the super-heavy nuclei \([\text{extensive calculations for nuclei around } Z\sim118 \text{ and } N\sim178]\); We expect obtaining theoretical details suited for experiment planning soon and will be interested in joining the experimental efforts.
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Thanks to: TetraNuc Collaboration [1]

I wish to thank for the support and participation in experiments as well as help in theoretical development the following Colleagues:

A. Góźdź, A. Dobrowolski - University of Lublin, Poland
N. Dubray - CEA, Bruyères-le-Châtel, F
N. Alahari, G. de France, M. Rejmund - GANIL, Caen, F
B. Lauss - ILL, Grenoble, F
N. Redon, Ch. Schmitt, O. Stézowski, D. Q. Tuyen - IPN, Lyon, F
A. Astier, G. Georgiev - CSNSM, Orsay, F

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J. Gerl - GSI, Darmstadt, Ge
R.P. Singh, S. Muralithar, R. Kumar, A. Jhingan, J.J. Das, R. K. Bhowmik - Inter-University Accelerator Centre, New Delhi 67, India
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D. Mengoni, F. Recchia, S. Aydin, R. Menegazzo, D. Bazzacco, E. Farnea, S. Lunardi, C. Ur - Dipartimento di Fisica and INFN, Sezione di Padova, Padova, Italy
Y. R. Shimizu - Kyushu University, Fukuoka, JP
P. Bednarczyk, B. Fornal, A. Maj, K. Mazurek - IFJ PAN, Krakow, Poland
J. Dobaczewski - University of Warsaw, Pl and JYFL, Jyväskylä, Fi
R.A. Bark, T.E. Madiba, T.D. Singo - iThemba LABS Physics Group;
D.G. Roux, J.F. Sharpey-Schafer, UWC, SA
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P. Regan - University of Surrey, UK
Thanks to: TetraNuc Collaboration [3]

Last but not least many thanks to an extremely dynamic contributions from our colleagues from the USA

L.L. Riedinger, Department of Physics, University of Tennessee, Knoxville, TN 37996
D.J. Hartley, Department of Physics, US Naval Academy, Annapolis, MD 21402
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W.D. Kulp - School of Physics, Georgia Institute of Technology, Atlanta, GA 30332
M.A. Riley - Department of Physics, Florida State University, Tallahassee, FL 32306

Jerzy DUDEK, University of Strasbourg, France
Tetrahedral Symmetry in Nuclei: Theory, Experimental Criteria
Thanks to: TetraNuc Collaboration [3]

Last but not least many thanks to an extremely dynamic contributions from our colleagues from the USA

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and was revived some 10 years later with a series of new articles

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A Basis for Octahedral Symmetry

Only special combinations of spherical harmonics may form a basis for surfaces with octahedral symmetry and only in even-orders:

Three Lowest Order Solutions:

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\lambda = 4 : & \quad \alpha_{40} \equiv o_4; \quad \alpha_{4,\pm4} \equiv \pm\sqrt{\frac{5}{14}} \cdot o_4 \\
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Namely:

- Tetrahedral group is a sub-group of the octahedral one !!!

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- Superposing octahedral and tetrahedral shapes leads to the final tetrahedral symmetry !!!
- Does realistic theory confirm these predictions?
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New Theory: Verification, Relation to Experiment

Experimental Facts: Comparison with Experiment

Example: Results with the HFB Solutions in RE

The HFB results for tetrahedral solutions in light Rare-Earth nuclei

\[ \alpha_{4,0} \equiv o_4, \quad \alpha_{4,\pm 4} \equiv -\sqrt{5/14} o_4 \]

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Octahedral deformations of the second order that are compatible with the tetrahedral deformation in $^{226}$Th with two Skyrme parameterisations

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![Diagram showing total energy vs. octupole deformation and single-nucleon energy vs. rotational frequency.](image-url)
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Spin-orientation probability at increasing spins of an octahedral nucleus
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Spin-Orientiation Probability

$I=10$, $n=1$; $E_{\text{rot}}=1.00000000000 (T2)$

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