U(3) and Pseudo-U(3) Symmetry of the Relativistic Harmonic Oscillator

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As is well-known, the non-relativistic spherical harmonic oscillator has degeneracies in addition to those due to rotational invariance. The energy spectrum depends only on the total harmonic oscillator quantum number \( N = 2n + \ell \), where \( n \) is the radial quantum number and \( \ell \) is the orbital angular momentum. Hence the states with \( \ell = N, N - 2, \ldots, 0 \) or 1 have the same energy. These degeneracies are produced by an U(3) symmetry [1]. This U(3) symmetry has been influential in connecting the shell model with collective motion [2]. Also the energy does not depend on the orientation of the spin and hence the non-relativistic harmonic oscillator has a spin symmetry as well.

Since relativistic models of nuclei are now so prevalent [3], we can ask if U(3) symmetry resides in the relativistic harmonic oscillator. Indeed the Dirac Hamiltonian, \( \hat{H} \), for which the scalar, \( V_S(\vec{r}) \), and vector, \( V_V(\vec{r}) \), potentials are equal and harmonic has been solved analytically and is invariant under a spin symmetry [4, 5]. Just as for the non-relativistic harmonic oscillator, the spherically symmetric relativistic harmonic oscillator energy spectrum depends only on the total harmonic oscillator quantum number \( N \), although the energy spectrum for the relativistic harmonic oscillator spectrum in general does not have a linear dependence on \( N \) as does the non-relativistic harmonic oscillator. This suggests that the relativistic harmonic oscillator does have an U(3) symmetry. If this is the case, the question is: what are the relativistic generators? In this talk we shall show that there is indeed a U(3) symmetry and we shall derive the generators which commute with \( \hat{H} \) [6].

Likewise, we show that a Dirac Hamiltonian, \( \tilde{H} \), for which the scalar, \( V'_{S}(\vec{r}) \), and vector, \( V'_{V}(\vec{r}) \), potentials are equal but opposite in sign and harmonic has a pseudospin symmetry and a pseudo-U(3) symmetry [4, 5].

References