High Q tunable LC resonator operating at cryogenic temperature

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Abstract

We present a step-motor based cryogenic tuning device for a high Q electrical resonator. The resonator can be tuned in the 145-175 Hz frequency range. Tuning is achieved by moving a superconducting core inside the 3 H superconducting coil of the resonator. The resonator intrinsic quality factor of 1.8×10^6 is found to be unaffected by the superconducting core and by the drive system. The noise of the resonator has been measured to be thermal. The device has been developed to improve sensitivity and bandwidth performance of cryogenic resonant gravitational wave detectors equipped with resonant capacitance transducers.

1 Introduction

The present resonant gravitational wave detectors [1, 2, 3, 4, 5] employ multimode networks for broadband impedance matching between the main mechanical resonator and sensing element (usually a few tons cylindrical bar) and the signal amplifier. In practice, those networks are constituted by one or two mechanical resonators connected to the bar (for a lumped model of a multimode detector see ref. [6]), with stepped down values of the masses;

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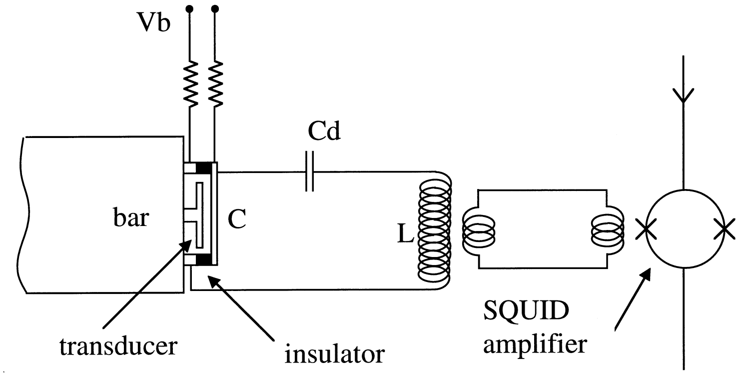

Figure 1: *Schematic of a capacitive transducer assembly on a resonant gravitational wave detector. The bar vibrations are converted into an electrical signal by a mushroom shaped capacitive transducer. The lowest symmetrical flexural mode of the transducer is tuned to the first longitudinal mode of the bar to form a system of two coupled mechanical oscillator. A third, electrical oscillator is composed by the transducer's capacitance C and by the inductance L of the primary of a matching transformer used to couple the device to a SQUID amplifier. The decoupling capacitance* $C_d \gg C$ *allows for charging the transducer with the bias voltage* V_b.

to benefit from higher mechanical amplification, the resonance frequencies of the network resonators, when uncoupled, are tuned to that of the bar.

In the detectors with resonant capacitive transducer [7] which use a dc SQUID as low noise amplifier, the matching network is composed in part by mechanical resonators and in part by electrical ones. All the operating detectors of this kind have two mechanical modes (bar and transducer modes) plus an electrical one (Fig. 1).

Giffard has shown that the energy sensitivity of a resonant detector is limited to the noise temperature of the amplifier. The limit is achieved when the detector is lossless [8]. Price [9] has subsequently shown that a multimode matching network allows for the achievement of the lossless sensitivity limit even if its modes have relatively low quality factors. Such a network also provides a larger bandwidth than that achieved with a single mode. Most of the improvement with a multimode system is obtained with a three or four modes system [10] additional modes only adding little benefit. Some two mechanical mode transducer have indeed been tested [11] but no one is employed in the operating cryogenic detectors. However, to achieve an optimal coupling among the resonators which constitute the antenna (bar plus matching network) it is necessary to tune their resonance frequencies within 1 Hz, an adjustment that is hard to preserve upon cooling of the detector.

Adjustment at room temperature followed by tests at low temperature can be iterated to achieve the tuning but, unfortunately, each iteration needs at least the time necessary for the whole thermal cycle (warm up and cooling) of the system which, for the present cryogenic detector, is approximately two months.

Usually in the detectors that use a resonant capacitive transducer and a dc SQUID, as that shown in Fig. 1, the electrical mode is kept weakly coupled to the mechanical ones [1]. That is the resonant frequency of the electrical mode is kept far enough from those of the mechanical ones not to spoil their high quality factors.

Recently a low loss superconducting coil has been developed [12] with quality factors comparable to those of the mechanical ones. Tuning of the electrical mode to the mechanical ones, though not fully equivalent to the addition of a third mechanical mode, still improve the performance of the detector almost as that addition would do [13]. In addition building a tuning system operating at cryogenic temperatures is easier for the electrical mode than for the mechanical modes [14].

It is worth noticing that apart from permitting the correction of unpredictable detuning taking place upon cooling, cryogenic tuning of the electrical mode can prove helpful in decoupling the electrical mode in case its quality factor results to be insufficient. Moreover, the possibility of adjusting the frequency of the electrical mode and hence of changing the position of poles and zeros in the impedance matching transfer function could be a useful to make the system stable, as it is known that the dc SQUID, because of its dynamic input impedance, can drive high quality factor systems producing instabilities [15, 16].

The main technical problems in the realization of a tuning device of this type are the preservation of the high quality factor and of a low, Nyquist limited, noise. Here we present a cryogenic tuning device that has indeed achieved these goals. The device has been set up in view of its use in the ultracryogenic gravitational wave detector AURIGA at the INFN Legnaro Laboratories [5].

2 Apparatus and experimental methods

The tuning device modulates the inductance of a superconducting coil similar to that now operating in the AURIGA detector. It consists of system driven by a stepper motor that moves two superconducting cylinders inside the superconducting coil. As a low magnetic field cannot penetrate the interior of a hard superconductor, the coil inductance is reduced when the cylinders

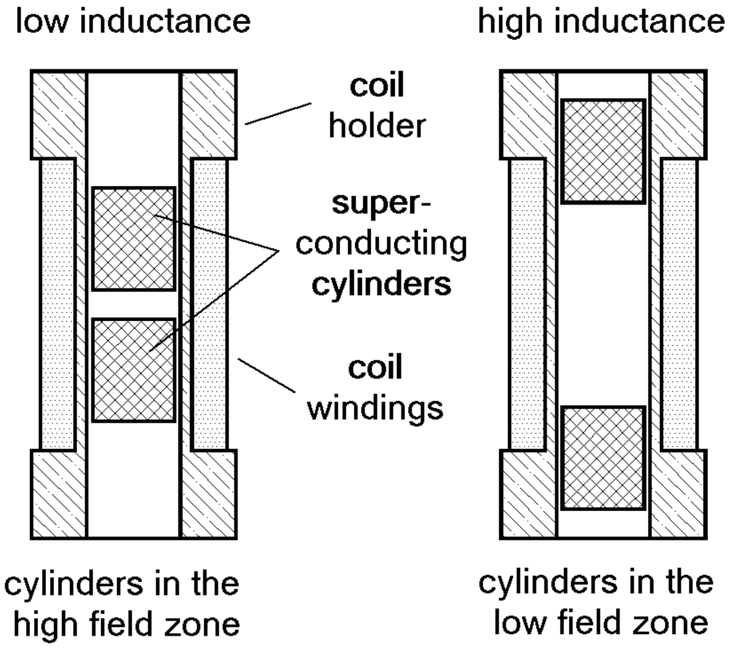

Figure 2: *Schematic of the coil inductance dependence from superconducting masses insertion.*

are moved from the coil end (low field zone) to the center (high field zone) as shown in Fig. 2. The stepper motor is operated only for the time requested to adjust the frequency value. A capacitor C is connected to the coil to form a high quality factor LC resonator, whose resonant frequency and quality factor are measured to detect the effects of the tuning device on the coil.

The low dissipation coil is made by winding $\approx 3\ km$ of 75 μm diameter NbTi wire coated with formvar to a total diameter of 100 μm onto a cylindrical teflon coil-holder. The total number of wire turns is ≈ 18000 in 35 layers. The coil has an internal diameter of 40 mm, external diameter of 70 mm and is 100 mm in height [12]. Its inductance is 5.9 H, that reduces to $L \simeq 3\,H$ inside the superconducting shield. The coil holder is 160 mm in height and has an internal hole of 32 mm diameter. Three teflon capacitors with a total capacity of $C = 390\ nF$ were connected to the coil to build the resonator. This capacity value fixes the oscillation frequency range $\nu = 1/(2\pi\sqrt{LC})$ around 150 Hz.

Though this frequency is different from that of the operating detectors ($\sim 900\ Hz$), it has been chosen because at 150 Hz the resonator dissipations are smaller and the system sensitivity to any dissipation added by the tuning system is then enhanced.

The system is vertically positioned as shown in Fig. 3, and the two identical superconducting cylinders hang from the same driving main shaft. The wires which hold the cylinders wind up in opposite directions, so that the upper cylinder is raised while the lower one is lowered. As the torques on the shaft due to the weight of the cylinders are balanced, a small commercial step motor [17] with a gear box designed to overcome only the friction force between the shaft and its bearings can be used. For compactness reason, the motor is fixed directly onto the coil holder [18]. In order to maximize the inductance variation, the total length of the two cylinders is equal to the coil length, and the coil-holder length allows the cylinders to be almost completely extracted from the coil. We have used two hollow superconducting cylinders in order to reduce weight and shaft friction. The cylinders are Pb7%Sn tubes with external diameter 31 mm and height 35 mm. Their wall thickness is 4 mm and their weight is 130 g each. Their external diameter is 1 mm less than their housing into the teflon coil-holder to deal with the differential thermal contraction between teflon and Pb7%Sn.

A special care is used to avoid spoiling the quality factor by eddy currents in any normal metal component of the setup. The cylindrical copper case housing of the LC resonator is electroplated with a 50 μm thick Pb7%Sn coating. This superconducting coating has the function both to avoid the eddy currents dissipation and to shield the resonator from the ambient magnetic field changes. The lead alloy with 7% of tin has been employed for its properties of hard superconductor, [19] in order to minimize the losses due to fluxons motion. All the metal parts used in the drive system are also made in superconducting material or are electroplated with Pb7%Sn and the wires which hold the cylinders are made in NbTi.

The commercial stepper motor radial tolerances are sufficient to allow free movement of the rotor all the way down to low temperature, provided that the bearing surface of the motor is washed with acetone to remove the oil at the surface [20]. The motor itself is 1.5 cm long and 2.5 cm in diameter. The rotor is made of a neodymium ferromagnet which has been magnetized with 48 adjacent regions of alternating polarity. The step angle of the motor is then 7.5 deg for our normal four stage (0+,+0,0-,-0) controller.

The driving system chassis is made of Pb7%Sn electroplated brass and supports the step motor at one side and the gear box at the other. The step motor is enclosed in a Pb7%Sn electroplated brass cap, with two small holes for the motor electrical connections and for the wires which hold the cylinders. The step motor drives a 2 mm diameter Nb shaft rotating into holes of the brass chassis. Nb shafts and holes have been accurately polished to reduce friction. 5 plastic gears are fixed on the shafts by small Nb pin to achieve a step angle reduction of $(25/12)^5$ at the main shaft. With our controller the room temperature torque exerted by the motor at the main shaft is $\sim 6 \times 10^{-3}\ N\,m$.

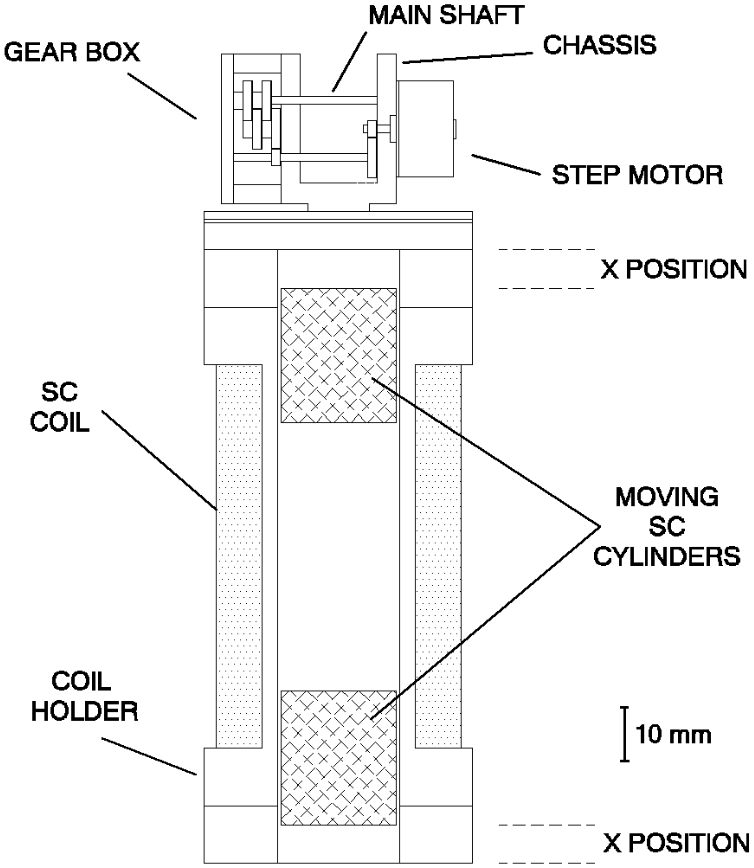

Figure 3: *Superconducting coil with the tuning system.*

Each cylinder hangs on a NbTi wire 0.12 mm diameter, wound onto a 4 mm diameter reel fixed on the main shaft. The stroke of each cylinder is about 45 mm, which corresponds to 6900 motor steps; the expected position resolution is then 6.5 μm. Qualitative tests performed at liquid nitrogen temperature have shown that the torque grows at low temperature, however the break limit of the NbTi wire, which corresponds to a torque of $\sim 10^{-2}\ Nm$ at the main shaft, is never exceeded. The upper stop at the end of the cylinder stroke is used to set the zero of the step counter.

All the inductance quality factor and noise measurements were carried out by immersing the cryostat in a liquid helium bath at 4.2 K. To ensure the thermal equilibrium of the resonator the case was filled with a small amount of He gas. All measurements were obviously performed with the step motor

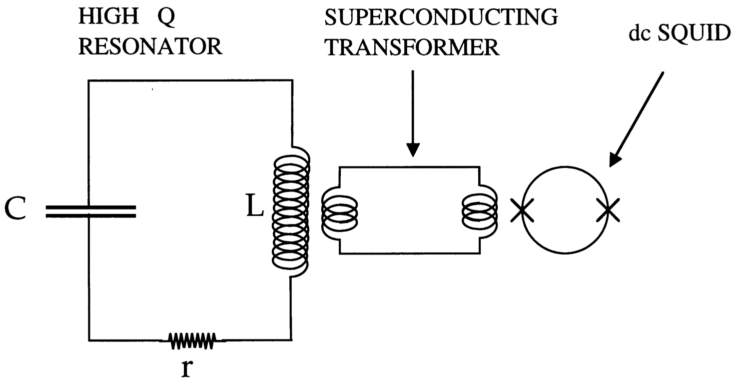

Figure 4: *Electrical measurement scheme. The resistance r represents the overall circuit dissipation and is related to the quality factor by* $Q = 2\pi f_0 L/r$.

off. The quality factor and the noise of the resonator were measured using a dc SQUID weakly coupled, $k^2 = 5 \times 10^{-3}$, to the superconducting coil by means of a superconducting transformer (Fig. 4). The weak coupling assures a negligible effect of the SQUID on the resonator.

The inductance and the Q factor of the LC resonator were measured from the resonance frequency f_0 and the time constant τ of the free decay respectively. To estimate possible added losses due to the tuning system, the measured quality factor of the resonator is compared with that of the resonator without the tuning system.

The noise of the resonator was measured [21] by sampling the quadrature components X_n and Y_n of the noise at the resonance frequency f_0 by means of a lock-in amplifier connected to the SQUID output. The lock-in time constant τ_{lk} is set so that $1/\tau_{lk}$ is slightly larger than the resonator bandwidth. X_n and Y_n were sampled at a sampling rate (0.0625 data/s) much faster than the decay time of the resonator, in order to easily recognize rare spurious non-gaussian events which are marked by a steep rise of the SQUID output. The data were then decimated to one every 2τ in order to obtain uncorrelated data sets. Exponential histograms of $R^2 = X_n^2 + Y_n^2$ were then produced. After having subtracted the component due to the SQUID broadband noise independently measured, the variance of the noise was then estimated from the exponential fit of the resulting histogram as in the standard analisys technique used in gravitational wave detectors. For reference, the resulting contribution has to be compared with the expected value of $G^2(kT/L)$ where G is the superconducting coil current to SQUID output voltage gain, k is the Boltzmann's constant and T the temperature. The value of G has been

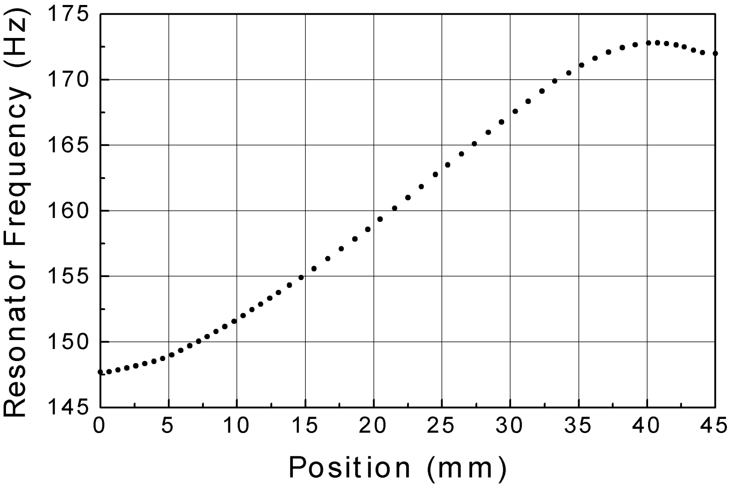

Figure 5: *Resonator frequency vs. cylinders position x.*

measured in a separate calibration run in which the superconducting coil was connected to a current generator.

3 Results

In Fig. 5 the measured resonance frequency is shown as a function of the distance x between the cylinders and the ends of the coil-holder. The figure shows that the resonator frequency can be adjusted anywhere in the range $147.8 - 172.8$ *Hz*.

The resonance frequency sensitivity to position is about 0.8 Hz/mm at $x = 22$ *mm*. It begins to saturate and approaches zero when the cylinders are near the coil holder ends, that is at $x = 0$, and when they are near the middle of the coil at $x = 45$ *mm*. The small sensitivity near $x = 0$ shows that the cylinders are almost extracted out of the coil and any further displacement does not change the inductance anymore.

The variation of the resonance frequency which corresponds to a single step is calculated to be ~ 5 mHz, but the frequency does not change uniformly when a series of single steps is applied. This is probably due to the static friction between the cylinders and the coil holder. The smallest resonance frequency variation that can be reproduced is 50 mHz, corresponding to 10 motor steps, so that the effective positioning resolution is about 65 μm. However the relative frequency stability, over a 70 hour measurements, is better than 2×10^{-6}. When the motor rotation direction is reversed the cylinders stop for 300 steps, because of the gear-box play. This inaccuracy

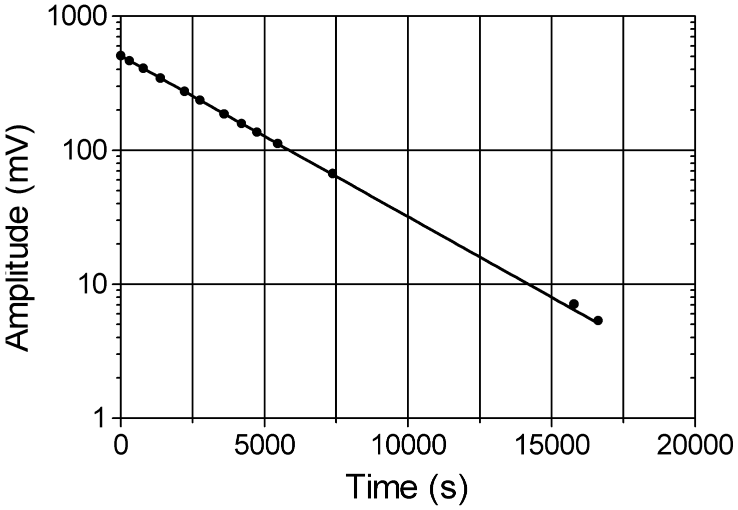

Figure 6: *Example of an exponential free decay of the resonator oscillation amplitude.*

does not affect the operation of the system within a tuning apparatus, as every frequency in the operating range can be reached within 50 mHz.

As shown in Fig. 5, the maximum relative inductance variation is $\simeq$ 30%; however, it is possible to increase considerably the relative inductance variation by realizing a coil in form of a thin solenoid (that is a solenoid with the winding thickness much less than its diameter) and by employing a superconducting core which can fill as much as possible the cross section of the coil. In our case the demand of a high inductance coil with reduced proportions leads to realize a coil with many winding layers and limits the maximum inductance variation. However this value is enough to allow the use of the device with that coil in the matching network of a gravitational wave detector like AURIGA.

The quality factor of the resonator with tuning device is $Q = 1.82 \pm .02 \times 10^6$ in the whole range of frequencies (Fig. 6). This value agrees with that measured in a separate test without the tuning device $Q = 1.83 \pm .02 \times 10^6$, showing that no degradation is brought about by the device.

In Fig. 7 we show the histogram of the squared amplitude of the noise R_n^2. The hystogram is in good agreement with a Boltzmann exponential with a temperature of (4.2 ± 0.3) K which agrees, within the errors, with the measured liquid helium bath temperature of 4.19 K. The six large amplitude data lying outside the statistics are all due to the single non gaussian event of unknown origin that occurred in 67 hour of continuous measurement.

From the above results we conclude that our tuning device: a) permits

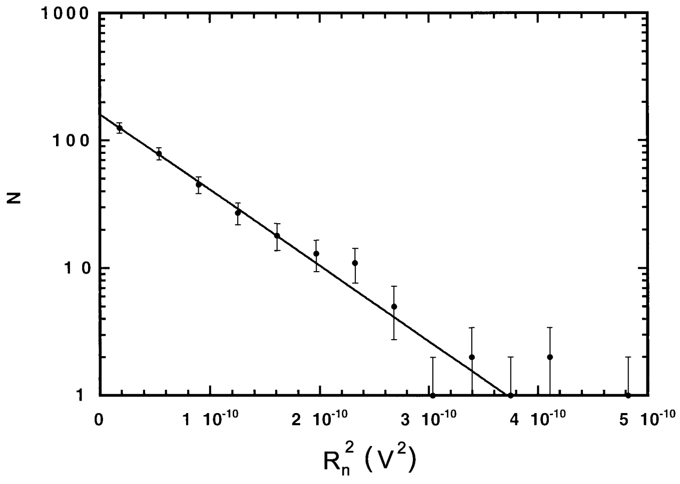

Figure 7: *Histogram of the number of samples of* R_n^2 *for each interval* $\Delta_{R_n^2} = 0.36 \times 10^{-10}\ V^2$.

a sufficient and stable frequency adjustement, b)does not affect the Q factor of the resonator, c) does not contribute any extra noise in excess of 0.3 K.

As it is easy to show that the ratio between the thermal noise and any other flux noise picked up by the superconducting coil is independent of the coupling with the SQUID, this result should hold also for the tight coupling case of the actual impedance matching transformer of gravitational wave detectors.

It is worth noticing that the mechanical isolation of the LC resonator in the present experiment is a few orders of magnitude worse than that of the real matching transformer mounted on the gravitational wave detector. The presently estimated upper limit of 0.3 K for the noise added by the device is then very likely an overestimate of the true noise that the device can attain once in real operation. We believe in conclusion that the tuning device we have presented here will successfully operate on the detector AURIGA.

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