Thermal noise in a high Q ultracryogenic resonator

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(Received 4 April 2005; accepted 16 May 2005; published online 22 June 2005)

A high Q electrical resonator based on a superconducting coil and a low loss capacitor has been realized and characterized at temperatures down to 60 mK. The resonance frequency is near 10 kHz, and the quality factor is higher than 10^5. The main purpose of the experiment is to test the feasibility of cooling to ultracryogenic temperatures the readout of the gravitational wave detector AURIGA, which is based on a high Q resonant electrical matching network. The resonator current noise, measured by a superconducting quantum interference device amplifier, scales with temperature in the full range 60 mK–4.2 K, showing that the system is thermal noise limited and its dissipating elements are well thermalized. Some implications of these results and possible applications are discussed. © *2005 American Institute of Physics.* [DOI: 10.1063/1.1947927]

I. INTRODUCTION

Electrical inductance-capacitance (LC) resonators operating in the kHz frequency range with quality factors up to 10^6 have been reported in literature.[1,2] These resonators are basically composed of low loss capacitors and superconducting coils enclosed in superconducting shields. Both dissipation and noise have been well characterized at liquid helium temperature (1.2–4.2 K).[3] Practical applications of such resonators include the characterization of the dynamic input impedance or the back action noise of superconducting quantum interference device (SQUID) amplifiers[4–6] and low noise field effect transistor (FET) amplifiers.[7]

Recently it has been recognized that tuning a high Q electrical resonant mode to the mechanical modes of a resonant gravitational wave detector,[8,9] which typically operates at about 1 kHz, leads to a significant increase of the detector bandwidth and sensitivity, provided that two requirements are fulfilled: the quality factor of the tuned electrical mode must be at least of the same order as that of the mechanical modes ($\simeq 10^6$), and its noise must be dominated by thermal noise. These conditions have been achieved in the new electromechanical readout of the AURIGA detector, which is currently operating in a long term run at liquid helium temperature.[10] The detector bandwidth has been increased with respect to the previous experimental setup from 1 to about 26 Hz, in good agreement with predictions,[9] and a record effective temperature $T_{\mathrm{eff}}=320\ \mu$K, that is the minimum detectable energy change expressed in temperature, has been reached. In the next experimental run, when the detector will be cooled to $\sim$100 mK, a further improvement of sensitivity by more than one order of magnitude is expected,[8] provided that the fundamental noise sources in the detector readout scale with temperature.

In particular, one must assume that the noise generated in the electrical mode is dominated by the thermal noise even at temperatures as low as 100 mK, and that the temperature of the dissipating elements effectively scales with the detector operating temperature. In principle these conditions can be obtained if all possible excess noise sources are kept negligible and if the time required for the thermalization is reasonably short with respect to the time scale of the experiment. Up to now, however, no experimental demonstration that these requirements are achievable has been reported. In this article we present a characterization of a high Q electrical resonator at ultracryogenic temperature. We present and discuss measurements of both noise and quality factor performed on the resonator in the temperature range 60 mK–4.2 K. In the final section we discuss the implications of the experimental results for the future sensitivity of the AURIGA detector and other possible applications of the resonator.

II. THEORETICAL MODEL

The electrical circuit model is shown in Fig. 1. Since the model has already been discussed,[3] we recall here only the main features. The LC resonator is modeled by lumped inductance and capacitance L and C. A lumped resistor r accounts for the resonator dissipation and for the thermal voltage noise source V_{th} with power spectral density $4k_BTr$. A voltage noise source V_{exc} with power spectral density S_{exc} accounts for other excess noise sources acting on the resonator. Possible excess noise mechanisms include vibrations of

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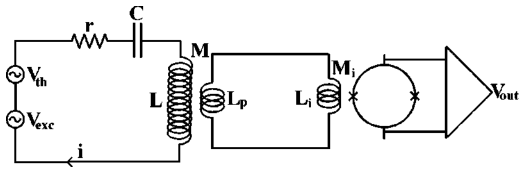

FIG. 1. Electrical circuit model of the LC resonator coupled to the SQUID amplifier, with the noise sources acting on the resonator.

the coil in the residual dc magnetic field trapped in the superconducting shields, fluctuating ac magnetic fields penetrating the shields and flux jumps in the shields.

The current flowing in the circuit is measured by a dc SQUID amplifier inductively coupled to the LC circuit by means of a weakly coupled pickup coil with inductance L_p and mutual inductance to the resonator coil M. The SQUID with its readout electronics is modeled as a current amplifier with input coil inductance L_i and current to voltage gain $A = M_i V_\phi$, where M_i is the mutual inductance between SQUID loop and input coil and V_ϕ is the flux to output voltage gain. We can define an overall gain factor $K = AM/(L_i + L_p)$, which converts the coil current i into the SQUID output voltage V_{out}. The factor $M/(L_i + L_p)$ is the current gain factor of the pickup circuit and can be interpreted as an effective coupling between LC resonator and SQUID. Its experimental value has been chosen high enough to resolve the LC resonator noise peak from the additive white noise floor of the SQUID, but small enough to keep negligible any back action effect on the resonator. In particular, we will neglect the SQUID back action noise,[5,6] the SQUID dynamic input impedance,[4] and the reduction of the inductance L due to the shielding effect of the pickup circuit.

Under these conditions the dynamics of the system is that of a simple resonator with quality factor and resonance frequency

$$\nu_0 = \frac{1}{2\pi\sqrt{LC}}, \tag{1}$$

$$Q = \frac{2\pi\nu_0 L}{r}. \tag{2}$$

The power spectral density of the resonator current noise has a Lorentzian shape and its integration over frequency yields the mean square current noise

$$\langle i^2 \rangle = \frac{k_B T}{L} + \frac{Q}{8\pi\nu_0 L^2} S_{\text{exc}}(\nu_0). \tag{3}$$

The first term on the right side of Eq. (3) represents the thermal noise, while the second term is the contribution of the excess voltage noise.

III. EXPERIMENTAL APPARATUS

A sketch of the experimental probe is shown in Fig. 2. The LC resonator and the SQUID amplifier are housed in a cylindrical copper box. The box and its top cover, also made of copper, are vacuum tight and assembled with a grease thread seal. The top cover of the box is attached to the mixing chamber of a dilution refrigerator unit. With respect to the indium seal, which requires space for flanges and bolt pattern, the grease thread seal requires less extra space on the box diameter. The seal is realized using two threaded rings. One of them, made of stainless steel 310, is threaded externally and glued with stycast epoxy on the box cover. The other, made of a high strength aluminum alloy (alumold), is threaded internally and glued with stycast epoxy on the top of the cylindrical box. The thread pitch is 0.5 mm, the diameter 66 mm and the total number of turns 15. Materials of the two rings have been chosen because they are machinable and nonmagnetic, and such that sealing is tightened from differential thermal contraction during the cooldown from room temperature to low temperature. Moreover, using different materials for the two rings decreases the probability of seizure. For the seal we used Dow Corning silicon vacuum grease because other nonsilicon products such as Apiezon greases tend to fragment at low temperature.

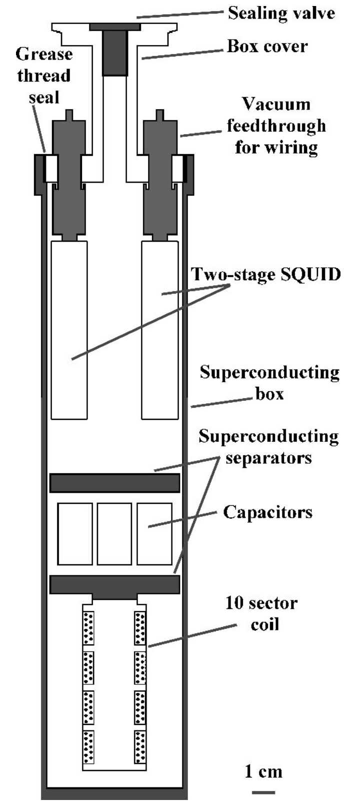

FIG. 2. Sketch of the experimental apparatus. For the sake of clarity some features have been simplified. In particular, the threaded rings for the box seal are not shown and the number of coil sectors has been reduced.

Two separators divide the internal space of the box into three sectors which house, respectively, from the bottom to the top, the coil, the capacitors and the SQUID. The box and the two separators are electroplated with a 40-μm-thick Pb 40% Sn coating,[11] with the exception of the upper part of the box near the threaded seal. This superconducting coating has the function of shielding the resonator, in particular the coil, from the ambient magnetic field and to avoid eddy current dissipation induced by the coil magnetic field, which would otherwise spoil the resonator quality factor.

The LC resonator is composed of a low loss capacitance and a superconducting coil. The capacitance is made by five commercial Teflon capacitors[12] connected in parallel, each with nominal capacitance 4.7 nF, for a total measured room temperature capacitance $C=23.5$ nF. The low dissipation coil is made by winding a formvar insulated 75-μm-diam NbTi wire (100 μm total diam) onto a cylindrical polyvinil chloride (PVC) holder. The coil holder, with 69 mm length and 20 mm external diameter, is divided into ten sectors, each 5 mm long. The coil is wound sequentially on each sector, three layers per sector, for a total number of 1377 turns. The estimated inductance of the coil is $L=10.7$ mH and the expected LC resonance frequency is 10037 Hz. Working at 10 kHz instead of 1 kHz allows a more compact experimental probe. Moreover, the vibrational noise at 10 kHz is much lower. Winding by sectors reduces the stray capacitance of the coil, which is formed between nearby layers. The contribution of the stray capacitance of the coil to the overall dissipation, expressed by the inverse of the quality factor, is given by

$$\frac{1}{Q_L} \cong \delta_L \frac{C_L}{C + C_L}, \tag{4}$$

where C_L and δ_L are the stray capacitance of the coil and the loss angle of the dielectric interposed between two nearby layers, which is basically constituted by the wire coating. It is thus convenient to reduce as much as possible C_L, or use coating with loss angle better than that of formvar, which has been measured of the order of 3×10^{-4} at $T=4.2$ K.[13] A coil with total length ℓ wound onto N_S identical piled sectors can be roughly approximated by the series of N_S coils with length ℓ/N_S, each one in parallel with its own stray capacitance. The stray capacitance of each sector scales with the length, so it is equal to C_ℓ/N_S, where C_ℓ is the stray capacitance of an equivalent single sector coil of length ℓ. As, in the multisector configuration, the capacitances of the sectors appear connected in series, the overall stray capacitance of the coil is expected to be C_ℓ/N_S^2. Thus winding by sectors allows one to obtain very small stray capacitance and, according to Eq. (4), very small dissipation, even without interposing Teflon layers between the coil layers to reduce the formvar dissipation, as described in a previous article.[2] In particular, for the coil described in this article we estimate $C_L\approx10$ pF, leading to $1/Q_L\sim10^{-7}$.

An auxiliary 20-mm-diam NbTi single turn coil is wound onto the coil holder in order to excite the LC resonator by means of an external signal.

The SQUID amplifier is a two-stage SQUID based on modified commercial chips, operated in closed loop mode. Details on this system can be found elsewhere.[5,14] The input coil inductance and its mutual inductance with the SQUID loop were separately measured $L_i=(1.66\pm0.02)$ μH and $M_i=(10.7\pm0.1)$ nH. The flux to voltage gain of the SQUID operated in closed loop mode was measured as $V_\phi=0.151$ V/ϕ_0 at 10 kHz, and the SQUID current to voltage gain is thus $A=M_iV_\phi=(7.8\pm0.1)\times10^5$ V/A. The SQUID flux noise is $S_{\phi\phi}=1.5\times10^{-12}$ ϕ_0^2/Hz at $T=4.2$ K and scales with temperature down to $T\simeq200$ mK.

The pickup coil is a ~3-mm-diam single loop of NbTi wire attached to the coil holder. The overall gain factor $K=AM/(L_i+L_p)=(9.2\pm0.1)\times10^3$ V/A was measured in a separate calibration run where the capacitor was disconnected and a known current was injected into the coil. Knowing A and K, the current gain factor of the pickup circuit $M/(L_i+L_p)=(1.18\pm0.02)\times10^{-2}$ has been calculated. On the basis of our past experience[4,5] we can estimate that, with such a small pickup gain factor, the contribution of the SQUID back action noise to $\langle i^2\rangle$ and the effect of the SQUID dynamic input impedance on Q are largely negligible. The reduction of the inductance L due to the shielding effect of the pickup circuit is negligible, too, so that we can apply the simple model described in Sec. II.

The refrigerator unit is a commercial continuous cycle ^{3}He–^{4}He dilution refrigerator[15] with nominal cooling power of 100 μW at 110 mK and a nominal unloaded base temperature of 18 mK. The resonator box is rigidly attached to the bottom of the refrigerator mixing chamber. The temperature of the probe is monitored by a calibrated germanium thermometer put on the box cover. With the probe attached to the mixing chamber the minimum measured temperature was about 60 mK. A heater placed on the mixing chamber allows changing the probe temperature in the range 60–500 mK. Higher temperatures are obtained by circulating only ^{3}He or ^{4}He in the refrigerator. Wiring from room temperature is progressively cooled by means of heat sink copper bobbins thermally anchored on the three main stages of the dilution refrigerator (1 K pot, still and mixing chamber). Wiring for the SQUIDs and the excitation coil enter the box through special cryogenic vacuum feedthroughs. Before attaching the probe to the refrigerator, exchange helium gas for the thermal sink of SQUID and resonator is introduced in the probe by means of a threaded sealing valve. As exchange gas we used either 0.8 atm of ^{3}He or 1 atm of ^{4}He at room temperature.

IV. MEASUREMENT METHODS

To measure the quality factor Q and the resonance frequency ν_0, the resonator is first excited and then let free to decay. The excitation is provided by a monochromatic signal sent to the excitation coil, with frequency ν_{lk} near the expected resonance frequency ν_0. The SQUID output signal is then sent to a lock-in amplifier with reference frequency ν_{lk} and the lock-in output magnitude and phase are sampled. The reference frequency ν_{lk} is then finely tuned to ν_0 within 1 mHz in order to minimize the phase shift rate, which is proportional to $(\nu_0-\nu_{\mathrm{lk}})$. Subsequently the resonator is excited again and ν_0 is evaluated from a linear fit of the sampled phases versus time. The quality factor is $Q=\pi\nu_0\tau$ where τ is the decay time constant evaluated from an exponential fit of the sampled magnitude versus time.

To measure the mean square current noise $\langle i^2\rangle$ of the unperturbed resonator we need to distinguish the narrowband process related to the resonator current from the white noise process related to the SQUID additive noise. To do this the SQUID output voltage is filtered by a lock-in amplifier with reference ν_{lk} and lock-in time constant τ_{lk}, in such way that

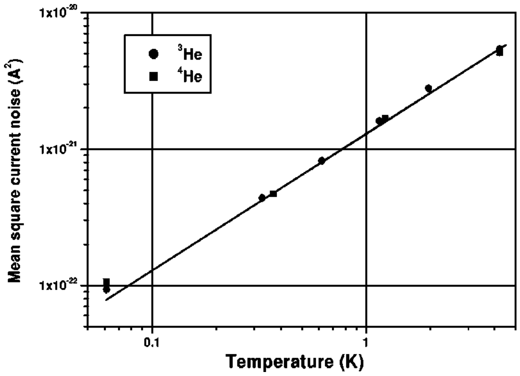

FIG. 3. Mean square current noise $\langle i^2 \rangle$ of the unperturbed resonator as function of the temperature T of the experimental probe, using both ^{3}He and ^{4}He as exchange gas. The error bar size is of the order of the point size. The predicted thermal noise $k_B T/L$ is represented by the straight line.

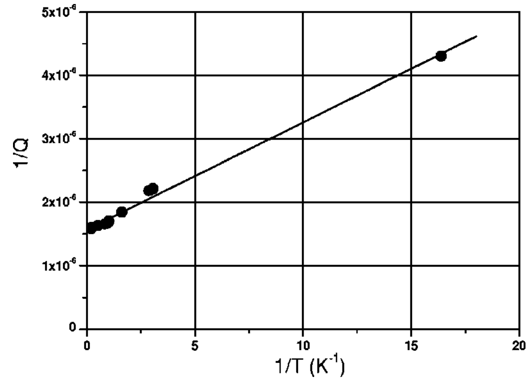

FIG. 4. Resonator dissipation, expressed by the inverse of the quality factor $1/Q$ as function of $1/T$. The straight line is a linear fit to the data.

$$|\nu_0 - \nu_{lk}| \ll \frac{1}{\tau_{lk}}. \tag{5}$$

Typically we set $\tau_{lk}=3$ s. The lock-in square magnitude $R^2 = X^2+Y^2$, where X and Y are the two lock-in quadrature phases, is then acquired. The sampling period is set of the order of τ. The expected distribution of the stochastic variable R^2 is exponential, provided that X and Y are Gaussian, and its mean value under the assumption (5) is given by

$$\langle R^2 \rangle = \langle R^2_{wb} \rangle + K^2 \langle i^2 \rangle \frac{\tau}{\tau + \tau_{lk}}. \tag{6}$$

The first term on the right side of Eq. (6) is the residual contribution of the SQUID wideband noise within the lock-in integration bandwidth while the second term is the resonator contribution corrected by the lock-in filtering.

To determine $\langle R^2 \rangle$ a histogram $N(R^2)$ is built, after at least 1000 samples are collected. A weighted exponential fit of the histogram, with errors given by $N^{1/2}$, is then performed with the expected exponential distribution

$$N(R^2) = N_0 \exp\left(-\frac{R^2}{\langle R^2 \rangle}\right) \tag{7}$$

and $\langle R^2 \rangle$ is extracted as fitting parameter. The residual wideband noise contribution $\langle R^2_{wb} \rangle$ is measured by repeating the same procedure with the same τ_{lk} well out of resonance, with the lock-in reference frequency shifted to $\nu_{lk}=\nu_0+30$ Hz. Typically $\langle R^2_{wb} \rangle$ was less than 10% of $\langle R^2 \rangle$.

The mean square resonator current noise $\langle i^2 \rangle$ is finally evaluated by inverting Eq. (6).

V. EXPERIMENTAL RESULTS AND DISCUSSION

The resonance frequency of the resonator has been measured as $\nu_0=10025$ Hz, quite close to the value calculated from the estimated values of L and C. The quality factor, of order 10^5, has shown a substantial dependance on temperature. We will discuss this secondary effect later.

The measurements of the resonator current noise $\langle i^2 \rangle$, using both ^{3}He and ^{4}He as exchange gas, are presented in Fig. 3. For each acquisition we have checked through a χ^2 test the consistency of the experimental R^2 distribution with the predicted one expressed by Eq. (7). Figure 3 demonstrates that the measured resonator noise is in good agreement with the predicted thermal noise over the full temperature range 60 mK–4.2 K, apart from a small excess noise at some temperature points. In particular, at $T=60$ mK we observe a significant excess noise equivalent to about ~20% of the thermal noise. This could suggest that the system had not yet reached the thermal equilibrium, but the excess noise is more likely to be related to external disturbances, since no special care has been taken to attenuate vibrational noise. In the near future we plan to improve the shielding of the resonator from dc magnetic fields in order to reduce the pickup of vibrational noise.

The observation of the thermal noise in the resonator proves that the dissipating elements are still essentially in thermal equilibrium with the bath even below 100 mK. On this basis we are entitled to expect the same behavior to be followed by the electrical readout of the AURIGA detector when operated at ultracryogenic temperature.[8] It must be noted, however, that the tuned high Q electrical network integrated in the AURIGA detector differs from the resonator described in this article in several points. In particular, in AURIGA the main capacitance is given by a capacitive transducer and the coil inductance must be about two orders of magnitude larger in order to tune the resonance frequency to about 1 kHz. The transducer is basically a vacuum-gap capacitor, and is expected to give only a small contribution to the total dissipation. Regarding the coil, a longer thermalization time is likely to be expected because of the larger dimension. However, this should not be a problem given the long operation time of the detector, of the order of several months. Finally, at 1 kHz the vibrational isolation of the resonator is much more difficult, but this task is already accomplished on the AURIGA detector by a high attenuation suspension system,[16] which is not used here.

Regarding the quality factor of the resonator, we have observed a rather unexpected dependance on temperature. Figure 4 shows the measured dissipation expressed by $1/Q$ as function of the inverse of temperature $1/T$. The evident increase of $1/Q$ at low temperature points to some kind of dissipation which grows roughly as $1/T$. In order to understand the origin of this unexpected behavior, which has not been pointed out in a previous work,[3] we have performed further quality factor measurements with a different resonator setup. For this purpose the resonator has been housed in the same experimental probe described by Bonaldi *et al.*,[3] which allows quicker tests with less liquid helium consumption in the temperature range 1.3–4.2 K. We have found that the temperature dependent dissipation is still present, and does not change significantly by using different coils with the same inductance or placing in parallel to the resonator an additional ~200 pF formvar capacitance to simulate an increased coil stray capacitance. This rules out the losses in the coil, in particular those related with the coil stray capacitance. Different behaviors are instead observed by changing the capacitor. In particular, the capacitors used by Bonaldi *et al.*[3] lead to a quality factor nearly independent of temperature, as was reported. Given these results, in the near future we plan to use homemade vacuum-gap capacitors, which should both improve the quality factor, because of reduced dielectric losses,[17] and give further indication on the origin of the temperature dependent dissipation.

The dependance of $1/Q$ on temperature has turned to be somehow useful in this experiment because it allows some rough considerations on the cooling efficiency. $1/Q$ follows very quickly the temperature variations down to about $T = 600$ mK, using both ^{4}He and ^{3}He as exchange gas, whereas significant delays are observed at lower temperature. For example when the mixing chamber is cooled from 600 to 60 mK the time required for $1/Q$ to approach a regime value is of the order of 1 day with ^{3}He exchange gas and 2 h with ^{4}He. The longer relaxation time can be interpreted in terms of a strong decrease of cooling efficiency with decreasing temperature. This is not surprising, because the rapid exponential drop of the vapor pressure of both ^{4}He and ^{3}He with decreasing temperature[18] makes for rapid vanishing of the heat transfer through the gas. Nevertheless, it is interesting to note that the cooling process seems much more efficient when ^{4}He is used instead of ^{3}He. The increased heat transfer with ^{4}He is likely to be related to the formation of superfluid film.

To conclude, we suggest some possible applications of the resonator. First it can be used to characterize the noise of SQUID amplifiers at ultracryogenic temperature and in particular to measure the back action noise.[5,6] When the resonator is optimally coupled to a SQUID the ratio of the $\langle i^2 \rangle$ back action contribution to the resonator thermal noise is given approximately by[19]

$$b = \frac{\langle i^2 \rangle_{\rm ba}}{\langle i^2 \rangle_{\rm th}} \approx \frac{\pi \nu_0 Q}{k_B T} \varepsilon_{\rm ba}, \tag{8}$$

where $\varepsilon_{\rm ba}$ is the SQUID back action noise spectral density expressed as energy per unit bandwidth in the input coil. For our resonator at $T=60$ mK and a quantum limited SQUID, for which $\varepsilon_{\rm ba} \simeq \hbar$, Eq. (8) yields $b \approx 1$. In other words, the thermal noise is so small that it should be possible to measure the back action of a quantum limited SQUID. Such a device is not available at present but a SQUID approaching the quantum limit at higher frequency has been reported.[20] In more general terms, one can observe the quantum limitations on the sensitivity of a linear measuring apparatus coupled to the resonator if the quality factor is at least[21]

$$Q \gtrsim \frac{k_B T}{h \nu_0}. \tag{9}$$

For our resonator at $T=60$ mK the condition (9) is satisfied. A quantum limited amplifier could permit one to detect changes of the resonator state as small as one single quantum, using well known techniques developed in the field of gravitational wave detection.[22] Furthermore, application of cold damping techniques[23] could permit cooling the macroscopic degree of freedom of the LC resonator down the quantum level, as has already been proposed for nanomechanical resonators.[24]

ACKNOWLEDGMENTS

The author thank Giuseppe Pinto for his skilled technical help. This work was in part supported by the Provincia Autonoma di Trento under the Project No. QL-READOUT.

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