Rotating Acoustic Black Holes and Superradiance in a non Linear Optical Cavity

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INTRODUCTION

The analogy between sound waves propagation in inhomogeneous fluids and light propagation in curved spacetime [1] could enable laboratory tests of relevant phenomena in black hole physics. So far, experimental proposals have been based on superfluid He and Bose-Einstein condensates (BECs) [2]. Recently however, it has been shown that, exploiting the relation between nonlinear optics and fluid dynamics, acoustic black holes can be created also in self-defocusing optical cavities. An optical field in self-defocusing media can be described in terms of a (2+1) “photon-fluid” on which linear perturbations, i.e. sound waves, experience an effective curved spacetime depending on the background flow (i.e. on the optical field profile). Since in an optical cavity the background flow is “pinned” by the driving field, the injection of a suitable optical vortex beam allows the generation of acoustic ergoregions and event horizons [3]. These results suggest the possibility to observe acoustic superradiance [4] from optical vortices in self-defocusing cavities.

OPTICAL VORTEX METRIC AND SUPERRADIANCE

We consider a self-defocusing cavity driven by the optical vortex beam \( E_d = \sqrt{\rho_d} \exp(i m \theta - 2i \pi r / r_0) \), where \( \rho_d \) is a constant optical intensity, \( m \) the vortex topological charge and \( r_0 \) an experimental parameter. Far from the vortex core, where diffraction effects are negligible, linear perturbations of the background field follow the massless Klein-Gordon equation \( \partial_\mu (g^{\mu \nu} \partial_\nu \psi) = 0 \) in a (2+1)-dimensional curved spacetime whose (covariant) metric reads

\[
g_{\mu \nu} = \begin{pmatrix}
-1 + \frac{\xi}{r^2} + \frac{m^2}{r^4} & 0 & \frac{m^2}{r} \\
0 & \left(1 - \frac{\xi}{r^2}\right)^{-1} & 0 \\
\frac{m^2}{r} & 0 & r^2
\end{pmatrix}
\]

(1)

where \( \xi = \frac{\lambda / n_2}{\sqrt{\rho_d} / r_0} \) plays the role of the healing length in BECs, \( \lambda \) is the optical wavelength, \( r_0 \) is the medium refractive index and \( n_2 \rho_d \) its optically-induced change [3]. Similarly to Kerr equator, the metric (1) has a coordinate singularity in correspondence of the event horizon, \( r_H = \xi^2 / r_0 \), and the radius of the ergosphere is given by the vanishing of its temporal component, i.e. \( r_E = \frac{1}{2} r_H + \sqrt{\frac{r_H^2}{4} + 4 m^2 / r_H r_0 / \xi^2} \).

We consider solutions of the Klein-Gordon equation with metric (1), \( \psi(t, r, \theta) = r^{-1/2} G(r^*) e^{i(\Omega t - m \theta)} \), where \( n \) is the wave winding number and \( r^* \) is a “tortoise coordinate” defined as \( dr^* = \left(1 - r_H / r\right)^{-1} dr \). The radial part of the sound wave, \( G(r^*) \), thus satisfies the equation

\[
\frac{d^2 G(r^*)}{dr^*} + \left( \frac{\Omega^2}{r} - \frac{m^2}{r^2} \right) G(r^*) + \left[ 1 - \frac{(m^2 - \xi)}{r^2} \right] G(r^*) = 0
\]

(2)

Note that \( r^* \) maps the horizon, \( r = r_H \), to \( r^* \to -\infty \), while \( r \to \infty \) corresponds to \( r^* \to +\infty \). Near the horizon and at the spatial infinity Eq. (2) reduces in both regions to harmonic oscillator equations which can be straightforwardly solved to give the asymptotic solutions

\[
G(r^*) = T e^{i(\Omega - n \Omega_H) r^*}, \quad r^* \to -\infty
\]

\[
G(r^*) = e^{i\Omega r^*} + K e^{-i\Omega r^*}, \quad r^* \to +\infty
\]

(3)

(4)

The wave (3) represents the only ingoing mode at the horizon, where \( T \) is the transmission coefficient, while in (4), the first and second term corresponds to an ingoing and a reflected wave with reflection coefficient \( K \) respectively. Since by Abel’s theorem, the Wronskian of these solutions is a constant independent of \( r^* \), from equality of the Wronskian at both asymptotics we get

\[
1 - |K|^2 = \left( \frac{\Omega - n \Omega_H}{\Omega} \right) |T|^2.
\]

(5)

This is precisely the superradiant amplification relation from rotating black holes in general relativity [5].

EXPERIMENTAL PROPOSAL AND NUMERICAL RESULTS

As a possible experimental system, we consider a 0.1-mm-long cavity with \( T = 10^{-2} \) filled with \(^{85}\text{Rb}\) vapor. The self-defocusing regime in such medium is obtained by detuning the laser to the red side of the hyperfine transition \( 5S_{1/2}(F = 2) - 5P_{3/2}(F = 3) \) \((\lambda \sim 780 \text{ nm})\) [6]. The detuning must be chosen in order to have the strongest nonlinearity compatible with an absorption lower than \( T \). A refractive index change \( n_2 \rho_d = 2 \times 10^{-6} \) leads to \( \xi = 275 \mu \text{m} \). Choosing the driving field \( E_d \) with \( r_0 = 100 \mu \text{m} \), the event horizon appears at \( r_H \approx 760 \mu \text{m} \). The location of the ergosphere, depending on the topological charge of the vortex \( m \), will be varied between \( r_E \approx 900 \mu \text{m} \) \((m = 4)\) and \( r_E \approx 1.3 \text{ mm} \) \((m = 10)\).

Keeping these parameters fixed, the reflection coefficients \( |K(\Omega)|^2 \) can be explicitly calculated by numerical integration of Eq. (2) with asymptotic solutions (3) – (4) and comparing the Fourier components of the incident and reflected wave.
Results for different values of the black hole rotation parameter $m$ and $n = 1, 2$ are reported in Fig. (1). As expected in the superradiant regime, $0 < \Omega < n\Omega_H$, we have $|\mathcal{R}|^2 > 1$ increasing for larger values of $m$ and being equal to 1 precisely at the characteristic frequency $n\Omega_H$. This is very similar to what observed in the case of massless fields in Kerr spacetime [7, 8] and, in the framework of analogue models, for the draining bathtub metric [9]. As in other acoustic black holes, superresonance is enormously efficient considering that typical amplifications of scalar fields in rotating astrophysical black holes are of the order of $0.2 - 0.3\%$ [7, 8]. In our case for $m = 10$, the maximum amplification is about 31% ($n = 1$) and 10% ($n = 2$). As the winding number $n$ with positive frequency waves, superradiance will occur only for positive $n$, i.e. for waves that are corotating with the black hole.

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