BEAM STABILITY IN SYNCHROTRONS WITH DIGITAL FILTERS IN THE FEEDBACK LOOP OF A TRANSVERSE DAMPER

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Abstract

The stability of an ion beam in synchrotrons with digital filters in the feedback loop of a transverse damper is treated. Solving the characteristic equation allows to calculate the achievable damping rates as a function of instability growth rate, feedback gain and parameters of the signal processing. A transverse feedback system (TFS) is required in synchrotrons to stabilize the high intensity ion beams against transverse instabilities and to damp the beam injection errors. The TFS damper kicker (DK) corrects the transverse momentum of a bunch in proportion to its displacement from the closed orbit at the location of the beam position monitor (BPM). The digital signal processing unit in the feedback loop between BPM and DK ensures a condition to achieve optimal damping. Transverse Feedback Systems commonly use digital FIR (finite impulse response) and IIR (infinite impulse response) filters for the signal processing. A notch filter is required to remove the closed orbit content of the signal and correct for the imperfect electric centre of the BPM. Further processing is required to adjust for the betatron phase advance between the beam pick-up (BPM) and the damper kicker (DK). Damping rates of the feedback systems with digital notch, Hilbert and all-pass filters are analysed in comparison with those in an ideal feedback system.

INTRODUCTION

Heavy ion beams of a high quality are required by many physicists for experimental studies. Gold ion beams are accelerated now in RHIC (BNL) [1], it is planed to accelerate lead ions in LHC [2]. Future accelerator facilities at GSI (FAIR project [3]) and JINR (NICA project [4]) are designed for acceleration of uranium beams. These facilities include a linear accelerator and several synchrotrons. For example, the CERN accelerator chain for ion beams consists of Linac – LEIR – PS – SPS – LHC. In the framework of the FAIR project the existing GSI accelerators serve as injectors for new synchrotrons SIS100 and SIS300. It is planned to build a booster as the injector for the Nuclotron operated now with a linac at JINR and to use the Nuclotron as the injector for a collider designed in the framework of the NICA project. It is clear that injection errors during the beam path from the linac to synchrotrons can lead to the undesirable growth of a beam emittance. It should be emphasised also that high intensity beams will be provided by these accelerators. The ultimate intensities after injection into the LHC will be about $4.8 \cdot 10^{10}$ ions for the ^{208}Pb^{82+} beam with an energy of 177 GeV/u. The peak intensities of particles after injection into the SIS100 will be about $5 \cdot 10^{11}$ for the ^{238}U^{28+} beam with an energy of 0.2 GeV/u. These intensities can lead to coherent transverse instabilities. Theoretical predictions for the instability rise time $\tau_{\rm inst}$ correspond to hundreds revolution periods $T_{\rm rev}$ of particles in the synchrotron. Therefore it is necessary to cure the transverse instabilities as well as to damp the transverse oscillations of the beam due to injection errors.

Transverse feedback systems (TFS) are used widely in synchrotrons for damping of coherent oscillations. The damping time $\tau_{\rm d}$ of TFS must be shorter the instability rise time $\tau_{\rm inst}$ to suppress instability: $\tau_{\rm d} < \tau_{\rm inst}$. In addition to that the damping time must be chosen to limit the emittance growth due to the beam injection errors. If $e_{\rm inj}$ is the maximum assumed amplitude of a beam deviation from the closed orbit due to displacement and angular errors at injection, then the relative emittance growth $\Delta\varepsilon/\epsilon$ is [5, 6]:

$$\frac{\Delta\epsilon}{\epsilon} = \frac{e_{\rm inj}^2}{2\sigma^2} F_{\rm a}^2\,; \qquad F_{\rm a} = \left(1 + \frac{\tau_{\rm dec}}{\tau_{\rm d}} - \frac{\tau_{\rm dec}}{\tau_{\rm inst}}\right)^{-1}, \quad (1)$$

where σ is the initial RMS beam size and $\tau_{\rm dec}$ is the beam decoherence time. Dependencies of the form factor $F_{\rm a}$ on the instability rise time $\tau_{\rm inst}$ for several values of the damping time $\tau_{\rm d}$ and the beam decoherence time $\tau_{\rm dec}$ are shown in Fig. 1. As rule $F_{\rm a} < 0.1$ is assumed that corresponds to $\tau_{\rm d} \approx 40 T_{\rm rev}$ for $\tau_{\rm inst} > 100 T_{\rm rev}$ and $\tau_{\rm dec} > 500 T_{\rm rev}$. The damping time $\tau_{\rm d} = 40 T_{\rm rev}$ is used commonly as the design specification of TFS for synchrotrons [7, 8].

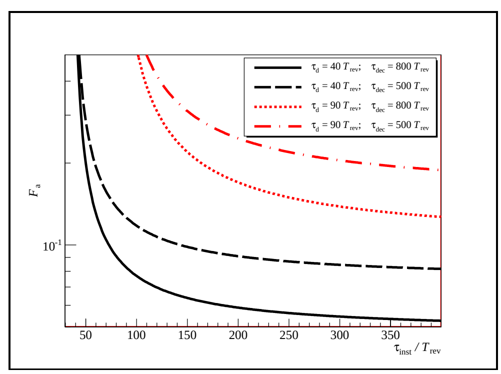

Figure 1: Dependencies of $F_{\rm a}$ on $\tau_{\rm inst}/T_{\rm rev}$.

BASIC DESCRIPTION

A classical transverse feedback system (see Fig. 2) consists of a beam position monitor (BPM), a damper kicker (DK) and an electronic feedback path with appropriate signal transmission from the BPM to the DK [9]. The damper

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kicker corrects the transverse momentum of a bunch in proportion to its displacement $x[n, s_\mathrm{P}]$ from the closed orbit at the BPM location s_P of the synchrotron's circumference C_0 at the n-th turn. The digital signal processing ensures the

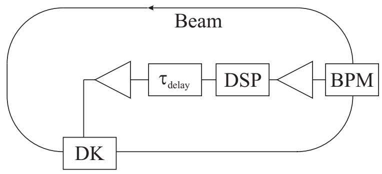

Figure 2: Layout of a classical transverse feedback system.

adjustment of the phase advance and the correction of the time of flight for optimum damping. The total delay τ_delay in the signal processing of the feedback loop from BPM to DK is adjusted to be equal to τ_PK, the particle flight of time from BPM to DK, plus an additional delay of q turns:

$$\tau_\mathrm{delay} = \tau_\mathrm{PK} + qT_\mathrm{rev}\,. \tag{2}$$

For the practical realization in a particle accelerator, we note that $q = 0$ or $q = 1$ are used [8].

Following the matrix description of the free oscillation of a particle in synchrotrons, the matrix equation for the bunch states at the BPM location s_P at the $(n+1)$ and n-th turns after a small kick by the DK is given by [10]

$$\begin{aligned}\widehat{X}[n+1, s_\mathrm{P}] &= \widehat{X}[n, s_\mathrm{P} + C_0] \\ &= \widehat{M_0}\widehat{X}[n, s_\mathrm{P}] + \widehat{B}\widehat{T}\Delta\widehat{X}[n, s_\mathrm{K}],\end{aligned} \tag{3}$$

where elements of the column matrix $\widehat{X}[n, s]$ are the bunch displacement $x[n, s]$ and the angle $x'[n.s]$ of its trajectory, $\widehat{M_0}$ is the revolution matrix, $\widehat{B}$ is an ordinary transfer matrix from the point $[n, s_\mathrm{K}]$ on the closed orbit at the DK location to the point $[n, s_\mathrm{P} + C_0]$ at the BPM position at the n-th turn, $\widehat{T}$ is the 2×2 matrix in which $T_{21} = 1$ and the other elements are zero. The first element of the column matrix $\Delta\widehat{X}$ in Eq. (3) is equal to the kick value $\Delta x'$

$$\Delta x'[n, s_\mathrm{K}] = S_\mathrm{K} V_\mathrm{out}[n]\,, \tag{4}$$

where S_K is the transfer characteristic of the damper kicker. The second element of the column matrix $\Delta\widehat{X}$ can be an arbitrary value due to the form of the matrix $\widehat{T}$. The output voltage $V_\mathrm{out}[n]$ of the feedback loop depends on the input voltage $V_\mathrm{in}[n, s_\mathrm{P}]$ at the BPM. In the general case of linear systems the output voltage can be written as follows:

$$V_\mathrm{out}[n] = u[n-q] \sum_{m=-\infty}^{n-q} h[n-m] V_\mathrm{in}[m, s_\mathrm{P}]\,, \tag{5}$$

where elements $h[m]$ are determined by the electronics in the feedback loop, $u[n]$ is the Heaviside step function and q corresponds to the number of turns for delay (see Eq.(2)). For a bunch injected at $n = 0$, the input voltage $V_\mathrm{in}[n, s_\mathrm{Pi}]$ depends on the bunch displacement at the BPM location:

$$V_\mathrm{in}[n, s_\mathrm{P}] = S_\mathrm{P} u[n] \left(x[n, s_\mathrm{P}] + \delta x\right), \tag{6}$$

where S_P is the BPM sensitivity and δx is a deviation of the BPM electric centre from a closed orbit. It should be emphasised that equations (3), (4), (5) and (6) correspond to the bunch-by-bunch feedback where the correction kick for a given bunch is computed based only on the motion of that bunch. Applying the bilateral Z-transform (see [11])

$$\mathbf{y}(z) = \sum_{n=-\infty}^{\infty} y[n] z^{-n}$$

in equations (3), (4), (5) and (6) we obtain from (3):

$$\widehat{\mathbf{X}}(z) = \frac{z\widehat{I} - \widehat{\mathbf{M}}^{-1} \det \widehat{\mathbf{M}}}{\det(z\widehat{I} - \widehat{\mathbf{M}})} \left(z \begin{pmatrix} x[0, s_\mathrm{P}] \\ x'[0, s_\mathrm{P}] \end{pmatrix} + \frac{z^{-q}\mathbf{K}(z)}{\sqrt{\hat\beta_\mathrm{P}\hat\beta_\mathrm{K}}} \widehat{B}\widehat{T} \begin{pmatrix} \delta x/(1-z^{-1}) \\ 0 \end{pmatrix} \right), \tag{7}$$

where $\widehat{I}$ is the identity matrix, the matrix $\widehat{\mathbf{M}}(z)$ is given by

$$\widehat{\mathbf{M}}(z) = \widehat{M}_0 + \frac{z^{-q}\mathbf{K}(z)}{\sqrt{\hat\beta_\mathrm{P}\hat\beta_\mathrm{K}}} \widehat{B}\widehat{T}\,, \tag{8}$$

the betatron amplitude function at the point s_P of the synchrotron's circumference is $\hat\beta_\mathrm{P} = \hat\beta(s_\mathrm{P})$, and $\hat\beta_\mathrm{K} = \hat\beta(s_\mathrm{K})$. The transfer function $\mathbf{K}(z)$ is determined by the system transfer function $H(z)$ of the electronics in the feedback loop in accordance with parameters $h[n]$ in (5):

$$\begin{aligned}\mathbf{K}(z) &= \sqrt{\hat\beta_\mathrm{P}\hat\beta_\mathrm{K}} S_\mathrm{P} S_\mathrm{K} H(z)\,, \\ H(z) &= \sum_{n=-\infty}^{\infty} z^{-n} h[n]\,.\end{aligned} \tag{9}$$

Consequently the bunch dynamics is determined by the poles z_k of $\widehat{\mathbf{X}}(z)$ which are roots of the characteristic equation:

$$\begin{aligned}\det\left(z_k\widehat{I} - \widehat{\mathbf{M}}(z_k)\right) &= z_k^2 - 2z_k \operatorname{Tr}\widehat{\mathbf{M}}(z_k) + \det\widehat{\mathbf{M}}(z_k) \\ &= z_k^2 - \left[2\cos(2\pi\widetilde{Q}) + z_k^{-q}\mathbf{K}(z_k)\sin(2\pi\widetilde{Q} - \psi_\mathrm{PK})\right] z_k \\ &\quad + 1 - z_k^{-q}\mathbf{K}(z_k)\sin\psi_\mathrm{PK} = 0\,,\end{aligned} \tag{10}$$

where $\widetilde{Q}$ is the beam tune, ψ_PK is the betatron oscillation phase advance from BPM to DK.

In the general case, $\widetilde{Q}$ is a complex function depending on z [12, 13]. The real part of $\widetilde{Q}$ is the number of betatron oscillations per turn: $\operatorname{Re}\widetilde{Q} = Q$. The imaginary part of $\widetilde{Q}$ is determined by the increment of the transverse instability: $2\pi \operatorname{Im}\widetilde{Q} = T_\mathrm{rev}/\tau_\mathrm{inst}$, where τ_inst is the transverse instability rise time.

The beam is stable if eigenvalues z_k from Eq.(10) lie inside the unit circle:

$$|z_k| < 1. \tag{11}$$

Damping rates of the coherent betatron oscillations are defined by the absolute value of z_k:

$$\frac{T_{\rm rev}}{\tau_k} = -\ln|z_k|\,, \tag{12}$$

where τ_k is the time constant of the betatron oscillation amplitude decay. Fractional parts $\{\mathrm{Re}\,\widetilde{Q}_k\}$ of the betatron frequency of a particle in presence of the transverse feedback system

$$\{\mathrm{Re}\,\widetilde{Q}_k\} = \frac{1}{2\pi}\arg(z_k) \tag{13}$$

are the fractional tunes ($-0.5 < \{\mathrm{Re}\,\widetilde{Q}_k\} \leq 0.5$).

If $|\mathbf{K}(z)| = 0$ then the solution of the Eq.(10)

$$z^{(0)}_{\pm} = \exp(\pm j2\pi\widetilde{Q}) \tag{14}$$

corresponds to the solution for frequencies of the betatron motion equation of a particle in synchrotrons. Let us assume that for small values of $|\mathbf{K}(z)|$ we can write:

$$z^{-q}\mathbf{K}(z) = g\exp(\mp j\varphi)\exp(\mp j2\pi q\widetilde{Q})\,, \tag{15}$$

where the gain $|g| \ll 1$ and the phase shift

$$\varphi = \arg\left(H(z^{(0)}_{-})\right) \tag{16}$$

of the feedback loop depend weakly on z, so that we can neglect dependences of g and φ on z in Eq.(10), and zero approximation from (14) can be used for g and φ at betatron frequencies. Let us assume also that the fractional part of the tune is not close to 0 or 0.5. In this case the solutions of Eq.(10) in the linear approximation with $|g| \ll 1$ are expressed by the formula:

$$z_{\pm} \approx \left(1 - \frac{g}{2}\exp(\pm j(\frac{\pi}{2} - \widetilde{\Psi}_{\rm PK}))\right)\exp(\pm j2\pi\widetilde{Q})\,, \tag{17}$$

where

$$\widetilde{\Psi}_{\rm PK} = \psi_{\rm PK} + 2\pi q\widetilde{Q} + \arg\left(H(z = \exp(-j2\pi\widetilde{Q}))\right). \tag{18}$$

Using definitions (12) and (13) the damping rates follow as

$$\frac{T_{\rm rev}}{\tau_{\pm}} \approx \frac{g\exp(\pm\,\mathrm{Im}\,\widetilde{\Psi}_{\rm PK})}{2}\sin(\mathrm{Re}\,\widetilde{\Psi}_{\rm PK}) \pm 2\pi\,\mathrm{Im}\,\widetilde{Q}\,, \tag{19}$$

and the fractional parts of tunes are:

$$\{\mathrm{Re}\,\widetilde{Q}_{\pm}\} \approx \pm\{Q\} \mp \frac{g\exp(\pm\,\mathrm{Im}\,\widetilde{\Psi}_{\rm PK})}{4\pi}\cos(\mathrm{Re}\,\widetilde{\Psi}_{\rm PK}). \tag{20}$$

Therefore the best damping of coherent transverse oscillations is achieved by optimally choosing the positions of BPM and DK yielding a phase advance of $\mathrm{Re}\,\widetilde{\Psi}_{\rm PK}$ equal to an odd multiple of $\pi/2$:

$$\mathrm{Re}\,\widetilde{\Psi}_{\rm PK} = \frac{\pi}{2}(2k+1), \tag{21}$$

where k is an integer. Hence the overall damping rate is:

$$\frac{T_{\rm rev}}{\tau} \approx \frac{g\exp(-\,\mathrm{Im}\,\widetilde{\Psi}_{\rm PK})}{2}\cos(\pi k) - 2\pi\,\mathrm{Im}\,\widetilde{Q} = \frac{T_{\rm rev}}{\tau_{\rm d}} - \frac{T_{\rm rev}}{\tau_{\rm inst}}\,,$$

where $\tau_{\rm d}$ is the damping time constant of the TFS without instability.

In the following transverse feedback systems satisfying the optimal conditions (2) and (21) are considered. We call the special case with $\varphi = 0$ and $q = 0$ hereafter the *ideal* transverse feedback system.

If $\widetilde{Q}$ depends weakly on z then the characteristic equation (10) with the feedback transfer function

$$z^{-q}\mathbf{K}(z) = ga_0z^{-q}H(z)$$

can be converted to a polynomial. It can be solved with the use of a root-finding algorithm or analytically for a polynomial of degree less than five. However, it is clear from (20) that $\{\mathrm{Re}\,\widetilde{Q}_k\} \approx \{Q\}$ for $|g| \ll 1$ in the case of (21). Therefore dependences of damping rates $|z_k|$ on gain g for the TFS with digital filters can be compared with those for the ideal TFS if a_0 is defined for $z_{\rm Q} = \exp(-j2\pi Q)$ such that

$$\begin{gathered} |a_0z_{\rm Q}^{-q}H(z_{\rm Q})| = 1, \\ a_0\sin\left(\arg\left(z_{\rm Q}^{-q}H(z_{\rm Q})\right) + \mathrm{Re}\,\psi_{\rm PK}\right) > 0. \end{gathered} \tag{22}$$

Hence the damping regime corresponds to $g > 0$. The calibration condition (22) will be used hereafter for all dependences of TFS damping parameters on gain g.

DIGITAL FEEDBACK SYSTEMS

Taking into account the final value theorem [11] and the solution (7) for $\widetilde{\mathbf{X}}(z)$ we can conclude that

$$\widehat{X}[\infty, s_{\rm P}] = \lim_{z\to 1}(1 - z^{-1})\widetilde{\mathbf{X}}(z) = 0 \quad \text{if} \quad \mathbf{K}(z=1) = 0\,. \tag{23}$$

Therefore as minimum a notch filter to suppress all the revolution harmonics (DC included) is required in the feedback loop. The magnitude of the difference signal from the BPM electrodes, after passing through the notch filter, is proportional to the bunch deviation from the closed orbit. The system transfer function of the notch filter is [11]:

$$H(z) = H_{\rm NF}(z) = 1 - z^{-1}. \tag{24}$$

It is clear from (24) that the notch filter changes the gain g and the phase φ of the open loop transfer characteristics. For example, if $Q = 6.73$ then $\{Q\} = -0.27$ and in accordance with (16) the phase φ is $\arg(H_{\rm NF}(z_{\rm Q})) = \varphi_{\rm NF} = 41.4°$. The gain $|H_{\rm NF}| = 2|\sin(\{Q\}\pi)| = 1.5$ can be adjusted by an amplifier a_0 in the feedback loop in accordance with (22). However, according to the approximation formula (19), the damping rates for the TFS with the notch filter still change due to the phase shift φ_{NF} resulting in slower damping than for the case of the ideal TFS.

The unwanted phase-shift φ_{NF} due to the notch filter can be compensated by a Hilbert filter [14] with the system transfer function

$$H_{\text{HF}}(z) = h_0 z^{-3} + h_1 z^{-2}(1 - z^{-2}) + h_3(1 - z^{-6}), \quad (25)$$

where

$$h_0 = \cos(\Delta\varphi), \; h_1 = -\frac{2}{\pi}\sin(\Delta\varphi), \; h_3 = -\frac{2}{3\pi}\sin(\Delta\varphi)$$

are the Hilbert transform impulse response coefficients.

The electric circuit of a feedback loop with the notch and Hilbert filters is shown in Fig. 3. The difference signal V_{in}

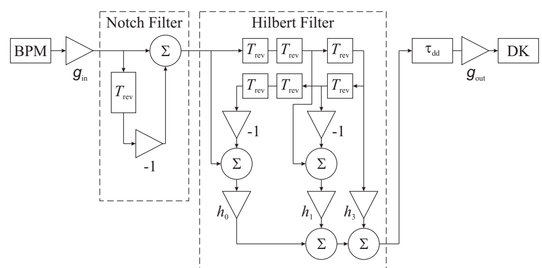

Figure 3: Block diagram of feedback loop with the notch and Hilbert filters.

from the electrodes of the beam position monitor (BPM) is amplified by front electronics with the gain g_{in}. Then the signal proceeds through the notch filter and the Hilbert filter. The synchronisation needed is adjusted by the digital delay τ_{dd}. The output voltage V_{out} on the damper kicker (DK) is supplied by the high power amplifier with the gain g_{out}. The notch filter has the standard configuration. It includes a one turn delay T_{rev}, an invertor and a summator. The Hilbert filter includes six one turn delays, four summators, two inverting amplifiers and three amplifiers h_0, h_1, h_3. For example, the phase shift needed for compensation of $\varphi_{\text{NF}} = 41.4°$ is obtained by using the Hilbert filter with $\Delta\varphi = -72.8°$.

The unwanted phase-shift φ_{NF} due to the notch filter can be compensated also by an all-pass filter [11] with a frequency-response magnitude that is constant but a phase advance which is variable and adjustable. The notch and Hilbert filters are FIR (finite impulse response) filters but the all-pass filter is IIR (infinite impulse response) filter. The transfer function of the first order an all-pass filter is

$$H_{\text{AF}}(z) = \frac{z^{-1} - a^*}{1 - az^{-1}}, \quad (26)$$

where a is a free filter parameter for the adjustment of the phase, and a^* denotes its complex conjugate. For example, the phase shift needed for compensation of $\varphi_{\text{NF}} = 41.4°$ is obtained by using the all-phase filter with $a = -0.501$.

The electric circuit of a feedback loop with the notch and all-pass filters is shown in Fig. 4. The all-pass filter includes a one turn delay T_{rev}, an inverting amplifier $(-1/a^*)$ in the non-recursive electric circuit, an amplifier a in the recursive electric circuit and two summators. An additional inverting amplifier $(-a^*)$ in the output electric circuit is ensuring $|H_{\text{AF}}| = 1$ for all frequencies independently on the filter parameter a. It allows to adjust phase shifts in the feedback loop by varying the parameter a but keeping the gain of the TFS constant.

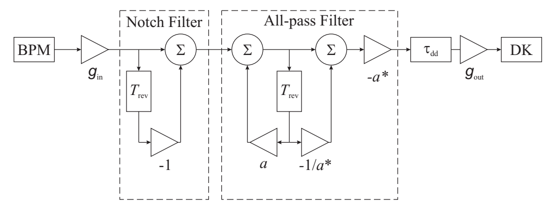

Figure 4: Block diagram of feedback loop with the notch and all-pass filters.

Dependences of damping rates $|z_k|$ on gain g for the ideal TFS, the TFS with notch and the TFS with notch and all-pass filters are shown in Fig. 5 (the tune of $Q = 6.73$ was used [15]). In case of the feedback loop with a notch

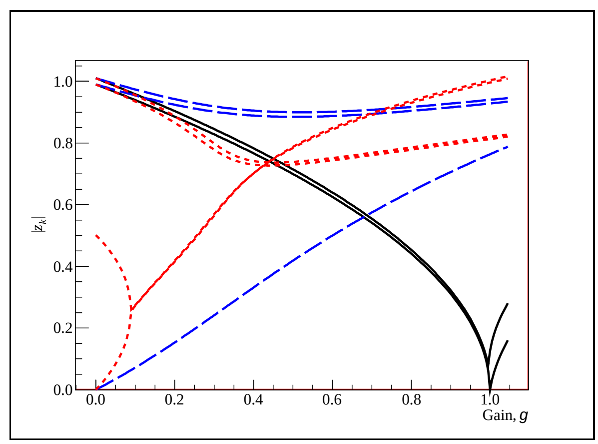

Figure 5: Dependences of damping rates $|z_k|$ on gain g for the ideal TFS (solid curves), for the TFS with the notch filter (dashed curves) and for the TFS with the notch and all-pass filters (dotted curves), parameter $a = -0.501$; shown is the case of the tune of $Q = 6.73$ and an assumed instability rise time of $\tau_{\text{inst}} = 100T_{\text{rev}}$.

filter only the Eq.(10) is a characteristic polynomial of the third degree. The characteristic equation (10) is a characteristic polynomial of the fourth degree in case of TFS with notch and all-pass filters. Therefore all dependences in Fig. 5 correspond to analytical solutions of Eq.(10). It is clear from Fig. 5 that the damping rates of the TFS with the notch filter are worse than the damping rates of the ideal TFS for all magnitudes of the feedback gains. However, for small gains $g \ll 1$ the characteristics of the TFS with the notch and all-pass filters coincide with the corresponding parameters of the ideal transverse feedback system if the phase shift of the notch filter was compensated by the all-pass filter with the parameter $a = -0.501$.

Dependences of overall damping rates $T_{\rm rev}/\tau$ on gain g for the ideal TFS and for feedback systems with digital notch, all-pass and Hilbert filters are shown in Fig. 6 in cases of optimal values for a and $\Delta\varphi$. Therefore the

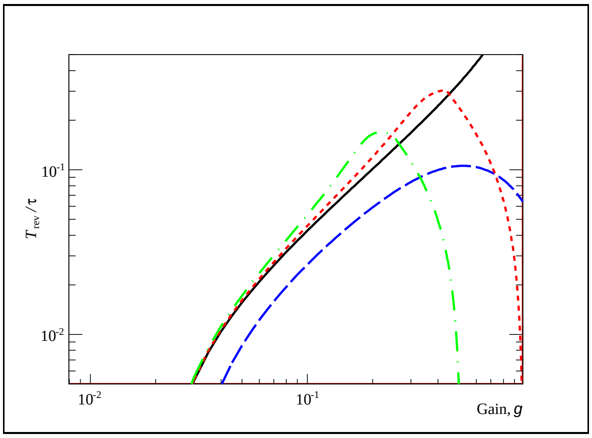

Figure 6: Dependences of overall damping rates $T_{\rm rev}/\tau$ on gain g for feedbacks: the ideal TFS (solid curve), with the notch filter (dashed curve), with the notch and all-pass filters for $a = -0.501$ (dotted curve), with the notch and Hilbert filters for $\Delta\varphi = -72.8°$ (dash-dotted curve); shown is the case of the tune of $Q = 6.73$ and an assumed instability rise time of $\tau_{\rm inst} = 100T_{\rm rev}$.

damping parameters of the ideal TFS can be obtained in the TFS with notch and all-pass or Hilbert filters for small gains. However the stability range is wider for TFS with the notch and all-pass filters. The gain g of TFS with the notch filter only must be in $\approx$1.3 times higher in the case of $\tau_{\rm d} = 40T_{\rm rev}$ than for TFS with the all-pass or Hilbert filter.

CONCLUSION

Following the analysis presented in this paper we can conclude that for small gains of the feedback loop the optimum damping characteristics of the ideal TFS can be restored in presence of a notch filter using a first order all-pass filter or a six order Hilbert filter with optimised parameters. Tuning the phase transfer characteristic of the all-pass or Hilbert filters in order to compensate the phase shift in the feedback loop caused by the notch filter we can obtain the optimal beam damping time. This possibility of tuning is an interesting feature and constitutes an advantage over a transverse damping system with a notch filter only.

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